

Probability Meets Algebra

1

(5 marks)

A bag contains n counters. 4 of the counters are red and the rest are blue.

Sam takes a counter at random and does not replace it. He then takes a second counter at random.

The probability that both counters are red is $\frac{2}{35}$.

(a) Show that $n^2 - n - 210 = 0$.

(b) Work out how many blue counters are in the bag.

$$P(\text{red, red}) = \frac{4}{n} \times \frac{3}{n-1} = \frac{12}{n(n-1)}$$

$$\frac{12}{n(n-1)} = \frac{2}{35} \Rightarrow 2n(n-1) = 420$$

$$n^2 - n = 210, \text{ so } \underline{n^2 - n - 210 = 0}$$

$$(n-15)(n+14) = 0, \text{ so } n = 15 \text{ (} n \text{ can't be negative)}$$

$$\text{Blue counters} = 15 - 4 = \underline{11}$$

2

(6 marks)

A box contains x red pens. There are 3 more green pens than red pens. There are no other pens.

Two pens are taken at random without replacement. The probability that both pens are green is $\frac{14}{39}$.

(a) Show that $17x^2 - 55x - 150 = 0$.

(b) How many pens were in the box at the start?

$$\text{Green} = x + 3, \text{ total} = 2x + 3$$

$$\frac{x+3}{2x+3} \times \frac{x+2}{2x+2} = \frac{14}{39}$$

$$39(x^2 + 5x + 6) = 14(4x^2 + 10x + 6)$$

$$39x^2 + 195x + 234 = 56x^2 + 140x + 84, \text{ so } \underline{17x^2 - 55x - 150 = 0}$$

$$(x-5)(17x+30) = 0, \text{ so } x = 5. \text{ Total} = 2(5) + 3 = \underline{13 \text{ pens}}$$

3

(6 marks)

A bag contains 6 yellow sweets and n purple sweets.

Mia takes two sweets at random without replacement. The probability that the two sweets are the **same colour** is $\frac{1}{2}$.

(a) Show that $n^2 - 13n + 30 = 0$.

(b) Find the two possible values of n .

$$P(YY) + P(PP) = \frac{30}{(n+6)(n+5)} + \frac{n(n-1)}{(n+6)(n+5)} = \frac{1}{2}$$

$$2(n^2 - n + 30) = (n+6)(n+5)$$

$$2n^2 - 2n + 60 = n^2 + 11n + 30, \text{ so } \underline{n^2 - 13n + 30 = 0}$$

$$(n-3)(n-10) = 0$$

$$\underline{n = 3 \text{ or } n = 10}$$

4

(5 marks)

In a jar, the ratio of orange sweets to lemon sweets is 2 : 3.

Ali takes two sweets at random from the jar and eats them. The probability that both sweets are lemon is $\frac{33}{95}$.

How many sweets were in the jar at the start?

$$\text{Let orange} = 2k, \text{ lemon} = 3k, \text{ total} = 5k$$

$$\frac{3k}{5k} \times \frac{3k-1}{5k-1} = \frac{33}{95} \Rightarrow \frac{3(3k-1)}{5(5k-1)} = \frac{33}{95}$$

$$285(3k-1) = 165(5k-1)$$

$$855k - 285 = 825k - 165, \text{ so } 30k = 120 \text{ and } k = 4$$

$$\text{Total} = 5k = \underline{20 \text{ sweets}}$$