

Simultaneous Equations: Special Cases

ANSWERS

1

Solve these equations, or show there is no solution or infinitely many.

$$(1) \quad x + 2y = 5$$

$$(2) \quad 3x + 6y = 11$$

$$(1) \times 3$$

$$(3) \quad 3x + 6y = 15$$

$$(3) - (2)$$

$$0 = 4$$

Answer no solution

The lines are parallel

2

Solve these equations, or show there is no solution or infinitely many.

$$(1) \quad 2x - 3y = 4$$

$$(2) \quad 8x - 12y = 16$$

$$(1) \times 4$$

$$(3) \quad 8x - 12y = 16$$

$$(3) - (2)$$

$$0 = 0$$

Answer infinitely many solutions

The lines are the same line

3

Solve these equations, or show there is no solution or infinitely many.

$$(1) \quad 5x + y = 3$$

$$(2) \quad 10x + 2y = 3$$

$$(1) \times 2$$

$$(3) \quad 10x + 2y = 6$$

$$(3) - (2)$$

$$0 = 3$$

Answer no solution

The lines are parallel

4

Solve these equations, or show there is no solution or infinitely many.

$$(1) \quad 3x - 4y = 2$$

$$(2) \quad -6x + 8y = -4$$

$$(1) \times 2$$

$$(3) \quad 6x - 8y = 4$$

$$(2) + (3)$$

$$0 = 0$$

Answer infinitely many solutions

The lines are the same line

5

Solve these equations, or show there is no solution or infinitely many.

$$(1) \quad x + 4y = 6$$

$$(2) \quad 3x + 10y = 16$$

$$(1) \times 3$$

$$(3) \quad 3x + 12y = 18$$

$$(3) - (2)$$

$$2y = 2$$

$$y = 1$$

$$(1) \quad x + 4 = 6$$

$$x = 2$$

Answer $x = 2, y = 1$

The lines cross at (2, 1)

6

Solve these equations, or show there is no solution or infinitely many.

$$(1) \quad y = 3x + 2$$

$$(2) \quad 9x - 3y = 4$$

$$\text{Substitute (1) into (2)}$$

$$9x - 3(3x + 2) = 4$$

$$9x - 9x - 6 = 4$$

$$-6 = 4$$

Answer no solution

The lines are parallel

7

Solve these equations, or show there is no solution or infinitely many.

$$(1) \quad 6x + 4y = 10$$

$$(2) \quad 9x + 6y = 15$$

$$(1) \times 3$$

$$(3) \quad 18x + 12y = 30$$

$$(2) \times 2$$

$$(4) \quad 18x + 12y = 30$$

$$(3) - (4)$$

$$0 = 0$$

Answer infinitely many solutions

The lines are the same line

8

Solve these equations, or show there is no solution or infinitely many.

$$(1) \quad x - 2y = 4$$

$$(2) \quad -3x + 4y = -11$$

$$(1) \times 3$$

$$(3) \quad 3x - 6y = 12$$

$$(2) + (3)$$

$$-2y = 1$$

$$y = -\frac{1}{2}$$

$$(1) \quad x + 1 = 4$$

$$x = 3$$

Answer $x = 3, y = -\frac{1}{2}$

The lines cross at $(3, -\frac{1}{2})$

Simultaneous Equations: Special Cases further practice

ANSWERS

9

How many solutions do these simultaneous equations have? Show your working.

$$(1) \quad x = 4 - 2y$$

$$(2) \quad 3x + 6y = 12$$

$$\text{Substitute (1) into (2)}$$

$$3(4 - 2y) + 6y = 12$$

$$12 - 6y + 6y = 12$$

$$12 = 12$$

$12 = 12$ is always true: the same line

Answer infinitely many solutions

10

Find the value of k so that $5x - 2y = 4$ and $kx - 6y = 1$ have no solution.

$$(1) \times 3: 15x - 6y = 12$$

The x and y terms match when $k = 15$, and $12 \neq 1$

Answer $k = 15$

11

Find a and b so that $3x + ay = 5$ and $6x + 8y = b$ have infinitely many solutions.

$$(1) \times 2: 6x + 2ay = 10$$

$2a = 8$, so $a = 4$. The same equation, so $b = 10$

Answer $a = 4, b = 10$

12

Sam solves $2x + 3y = 6$ and $4x + 6y = 12$. His working ends $0 = 0$, so he writes $x = 0, y = 0$. What has Sam done wrong? Give two solutions.

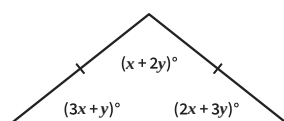
$0 = 0$ is always true: infinitely many solutions, every point on the line. $x = 0, y = 0$ does not fit: $0 \neq 6$

for example $x = 3, y = 0$ and

Answer $x = 0, y = 2$

13

This isosceles triangle is not drawn accurately. Find x, y and the three angles.



$$3x + y = 2x + 3y, \text{ so } x - 2y = 0$$

$$\text{Angles add to } 180^\circ: 7x + 4y = 180$$

$$(1) \times 2, \text{ then add: } 9x = 180$$

$$x = 20, y = 10; 70^\circ$$

Answer $70^\circ, 40^\circ$

14

£8000 is split between two accounts, both paying simple interest for one year. One pays 2% and the other 4%. Together they earn £250. How much is in each account?

$$\text{Let } a = \text{£ at } 2\%, b = \text{£ at } 4\%$$

$$a + b = 8000, 2a + 4b = 25000$$

$$(1) \times 2, \text{ then } (2) - (3): 2b = 9000$$

$$b = 4500, a = 3500$$

Answer 2%: £3500, 4%: £4500

15

Solve $47x + 23y = 209$ and $23x + 47y = 281$. Add the whole equations, then subtract them. What do you notice?

$$\text{Add: } 70x + 70y = 490, \text{ so } x + y = 7$$

$$\text{Subtract: } 24x - 24y = -72, \text{ so } x - y = -3$$

$$x - y = -3$$

$$\text{Then } 2x = 4.$$

$$\text{Check: } 94 + 115 = 209 \checkmark$$

Answer $x = 2, y = 5$

16

Write your own pair of simultaneous equations that the method in Q15 works on. Solve them that way. Explain why it works.

The coefficients swap places, so adding makes them equal:

$$8x + 8y = 24, \text{ so } x + y = 3.$$

Subtracting leaves them opposite:

$$2x - 2y = 2, \text{ so } x - y = 1. x = 2, y = 1$$

for example $5x + 3y = 13,$

Answer $3x + 5y = 11$