

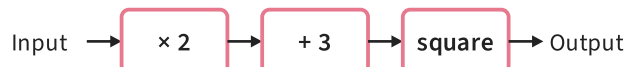
1 Function machines

Here is a function machine.

a) Write the output in terms of the input x .

Expand your answer.

b) Show that the output is never 9 times the input.



a) $(2x + 3)^2 = 4x^2 + 12x + 9$

b) $4x^2 + 3x + 9 = 0$

$b^2 - 4ac = 9 - 144 < 0$: no solutions

2 Function machines

$f(x) = x^2 - 2x$

a) Show that $f(x + 1) = x^2 - 1$

b) Solve $f(2x) = 2f(x) + 12$. Give exact answers.

a) $x^2 + 2x + 1 - 2x - 2 = x^2 - 1$

b) $4x^2 - 4x = 2x^2 - 4x + 12$

$2x^2 = 12$, $x = \pm\sqrt{6}$

3 Inverse functions

$f(x) = kx + 6$ and $f^{-1}(10) = 2$

a) Find k , then find $f^{-1}(x)$.

b) Solve $f^{-1}(3x) = f(x)$.

a) $f(2) = 10$, so $k = 2$, $f^{-1}(x) = \frac{x - 6}{2}$

b) $3x - 6 = 4x + 12$

$x = -18$

4 Inverse functions

$f(x) = 4 + \sqrt{x - 1}$ for $x \geq 1$

a) Find $f^{-1}(x)$.

b) Explain why $f^{-1}(2)$ does not exist.

a) $\sqrt{x - 1} = y - 4$, $x - 1 = (y - 4)^2$

$f^{-1}(x) = (x - 4)^2 + 1$

b) $\sqrt{} \geq 0$, so $f(x) \geq 4$ and is never 2

Functions revision mat answers

5 Inverse functions

$f(x) = \frac{3x + 1}{x - 3}$

a) Show that $f^{-1}(x) = f(x)$

b) Hence write down $ff(7)$.

a) $yx - 3y = 3x + 1$, $x(y - 3) = 3y + 1$

$f^{-1}(x) = \frac{3x + 1}{x - 3}$, the same

b) f undoes itself: $ff(7) = 7$

6 Composite functions

$f(x) = 2x - 3$ and $g(x) = \frac{1}{x}$

a) Write $fg(x)$ and $gf(x)$ as single fractions.

b) Solve $fg(x) = gf(x)$.

a) $fg(x) = \frac{2 - 3x}{x}$, $gf(x) = \frac{1}{2x - 3}$

b) $(2 - 3x)(2x - 3) = x$, $6x^2 - 12x + 6 = 0$

$(x - 1)^2 = 0$, $x = 1$

7 Composite functions

$f(x) = x^2 + 4x$ and $g(x) = x - 2$

a) Show that $fg(x) = x^2 - 4$

b) Solve $gf(x) = fg(x)$.

a) $(x - 2)^2 + 4(x - 2) = x^2 - 4x + 4 + 4x - 8$

b) $x^2 + 4x - 2 = x^2 - 4$

$4x = -2$, $x = -0.5$

8 Composite functions

$f(x) = ax + b$ and $ff(x) = 9x - 8$

a) Find the two possible pairs of values of a and b .

b) Solve $ff(x) = x$.

a) $a^2 = 9$ and $ab + b = -8$

$a = 3$, $b = -2$ or $a = -3$, $b = 4$

b) $9x - 8 = x$, $x = 1$