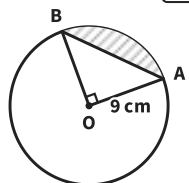


1 Parts of a circle

- a) A sector of a circle of radius 9 cm has an arc of length 4π cm. Find the angle of the sector.
- b) Name the shaded region. Find its exact area.



Not drawn accurately

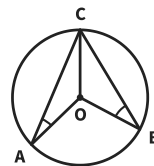
$$a) \frac{\theta}{360} \times 18\pi = 4\pi, \theta = 80^\circ$$

$$b) \text{a segment: } \frac{81\pi}{4} - \frac{81}{2} \text{ cm}^2$$

2 Angle at the centre

O is the centre. Angle $OAC = 23^\circ$ and angle $OBC = 31^\circ$.

- a) Find angle AOB.
- b) Now let angle $OAC = x$ and angle $OBC = y$. Prove that angle $AOB = 2 \times \text{angle } ACB$.



Not drawn accurately

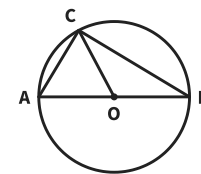
$$a) \angle ACB = 23 + 31 = 54^\circ \text{ (isosceles), } \angle AOB = 108^\circ$$

$$b) \angle AOC = 180 - 2x, \angle BOC = 180 - 2y$$

$$\angle AOB = 2x + 2y = 2 \times \angle ACB$$

3 Angle in a semicircle

- a) AB is a diameter, centre O. Prove that angle $ACB = 90^\circ$.
- b) A circle has diameter AB, with $A(-3, 1)$ and $B(5, 7)$. Show that $C(4, 0)$ is on the circle.



$$a) \angle OAC = \angle OCA = a, \angle OBC = \angle OCB = b \text{ (radii)}$$

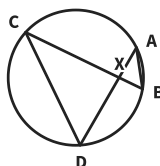
$$2a + 2b = 180, \angle ACB = a + b = 90^\circ$$

$$b) \text{gradients } AC, BC: -1 \div 7 \times 7 = -1, \text{ so } \angle ACB = 90^\circ \text{ and } C \text{ is on it}$$

4 Angles in the same segment

Chords AD and BC cross at X.

- a) Prove that triangles AXB and CXD are similar.
- b) $XA = 4$ cm, $XD = 9$ cm and $XB = 3$ cm. Find XC.



Not drawn accurately

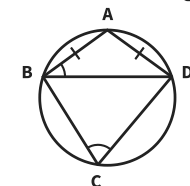
$$a) \angle BAD = \angle BCD \text{ and } \angle ABC = \angle ADC \text{ (angles in the same segment), so } \triangle AXB \sim \triangle CXD$$

$$b) \frac{XA}{XC} = \frac{XB}{XD} : XC = \frac{4 \times 9}{3} = 12 \text{ cm}$$

Circle geometry revision mat answers

5 Cyclic quadrilaterals

- a) ABCD is a cyclic quadrilateral with $AB = AD$, angle $BCD = 3x^\circ$ and angle $ABD = (x + 12)^\circ$. Find x.
- b) A cyclic quadrilateral's angles, in order, are in the ratio $2 : 3 : 7 : k$. Find k.



Not drawn accurately

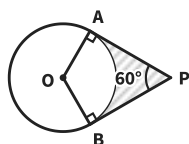
$$a) \angle BAD = 180 - 3x, \angle ABD = 1.5x = x + 12, x = 24$$

$$b) 2 + 7 = 3 + k, k = 6$$

6 Tangents

PA and PB are tangents to the circle, centre O, radius r.

- a) Show that $OP = 2r$.
- b) When $r = 6$ cm, find the exact area of the shaded region.



Not drawn accurately

$$a) \angle OAP = 90^\circ, \angle OPA = 30^\circ, OP = r \div \sin 30^\circ = 2r$$

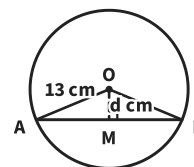
$$b) \text{kite } 2 \times \frac{1}{2} \times 6 \times 6\sqrt{3} = 36\sqrt{3}, \text{ sector } 12\pi$$

$$36\sqrt{3} - 12\pi \text{ cm}^2$$

7 Chords

O is the centre. The chord AB is d cm from O and is $2(d + 7)$ cm long.

- a) Find d.
- b) Find angle AOB to 1 decimal place.



Not drawn accurately

$$a) d^2 + (d + 7)^2 = 13^2, d^2 + 7d - 60 = 0$$

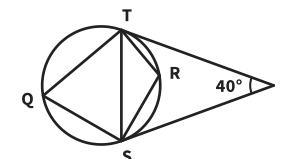
$$(d + 12)(d - 5) = 0, d = 5$$

$$b) MB = 12, \angle AOB = 2 \tan^{-1}\left(\frac{12}{5}\right) = 134.8^\circ$$

8 Alternate segment

PT and PS are tangents to the circle. Q and R are on the circle. Give reasons.

- a) Find angle TQS.
- b) Find angle TRS.



Not drawn accurately

$$a) PT = PS, \text{ so } \angle PTS = (180 - 40) \div 2 = 70^\circ$$

$$\angle TQS = 70^\circ \text{ (alternate segment)}$$

$$b) \angle TQR \text{ is cyclic: } 180 - 70 = 110^\circ$$