

1 Equation or identity?

- a) Show that $(2x-1)^2 - (x-2)^2 \equiv 3(x^2-1)$
b) Explain why $x(x+4) = (x+2)^2$ has no solutions.

$$\begin{aligned} \text{a) } & 4x^2 - 4x + 1 - (x^2 - 4x + 4) \\ & = 3x^2 - 3 = 3(x^2 - 1) \\ \text{b) } & x^2 + 4x = x^2 + 4x + 4 \\ & \text{so } 0 = 4, \text{ never true} \end{aligned}$$

2 Identities: finding unknowns

- a) $(ax+3)(x-b) \equiv 2x^2 - 7x - 15$. Find a and b .
b) $3(x-p)^2 + q \equiv 3x^2 - 12x + 7$. Find p and q .

$$\begin{aligned} \text{a) } & a = 2, -3b = -15: b = 5 \\ & \text{check: } -10x + 3x = -7x \\ \text{b) } & -6p = -12: p = 2, 12 + q = 7: q = -5 \end{aligned}$$

3 Simplifying algebraic fractions

Simplify fully

a) $\frac{6x^2 - 7x - 3}{9x^2 - 1}$

b) $\frac{x^2 - 2x - 8}{4 - x}$

$$\begin{aligned} \text{a) } & \frac{(2x-3)(3x+1)}{(3x-1)(3x+1)} = \frac{2x-3}{3x-1} \\ \text{b) } & \frac{(x-4)(x+2)}{-(x-4)} = -(x+2) \end{aligned}$$

4 Multiplying and dividing

Simplify fully

a) $\frac{x^2-9}{2x^2+5x-3} \div \frac{x^2-6x+9}{4x^2-1}$

b) $(1 + \frac{2}{x}) \div \frac{x^2-4}{x^2}$

$$\begin{aligned} \text{a) } & \frac{(x-3)(x+3)}{(2x-1)(x+3)} \times \frac{(2x-1)(2x+1)}{(x-3)^2} = \frac{2x+1}{x-3} \\ \text{b) } & \frac{x+2}{x} \times \frac{x^2}{(x-2)(x+2)} = \frac{x}{x-2} \end{aligned}$$

Algebraic fractions and algebraic proof *revision mat answers*

5 Adding and subtracting

Write each as a single fraction in its simplest form.

a) $\frac{2}{x+3} + \frac{x-1}{x^2+2x-3}$

b) $\frac{x}{x-3} - \frac{18}{x^2-9}$

$$\begin{aligned} \text{a) } & \frac{2}{x+3} + \frac{1}{x+3} = \frac{3}{x+3} \\ \text{b) } & \frac{x(x+3)-18}{(x-3)(x+3)} = \frac{(x+6)(x-3)}{(x-3)(x+3)} = \frac{x+6}{x+3} \end{aligned}$$

6 Algebraic fractions: equations

Solve $\frac{2x}{x+1} - \frac{1}{x-2} = 1$

Give your answers in surd form.

$$\begin{aligned} 2x(x-2) - (x+1) &= (x+1)(x-2) \\ 2x^2 - 5x - 1 &= x^2 - x - 2 \\ x^2 - 4x + 1 &= 0, x = \frac{4 \pm \sqrt{12}}{2} \\ x &= 2 \pm \sqrt{3} \end{aligned}$$

7 Algebraic proof

- a) Prove that the sum of the squares of two consecutive odd numbers is 2 more than a multiple of 8.
b) Prove that the mean of four consecutive whole numbers is never a whole number.

$$\begin{aligned} \text{a) } & (2n+1)^2 + (2n+3)^2 = 8n^2 + 16n + 10 \\ & = 8(n^2 + 2n + 1) + 2 \\ \text{b) } & \frac{4n+6}{4} = n + 1.5, \text{ never whole} \end{aligned}$$

8 Algebraic proof: true or false?

- a) True or false: when n is odd, $n^2 - 1$ is a multiple of 8. Prove your answer.
b) Is $x^2 - 6x + 11$ ever negative? Prove your answer.

$$\begin{aligned} \text{a) true: } & (2n+1)^2 - 1 = 4n(n+1) \\ & n(n+1) \text{ is even, so } 8 \text{ is a factor} \\ \text{b) no: } & (x-3)^2 + 2 \geq 2 \end{aligned}$$