

1 Equation or identity?

P $4(x-3) = 4x - 12$ Q $3x + 2 = 2x + 9$ R $(x+1)^2 = x^2 + 1$

a) Which of P, Q and R are identities?

b) Show that $2(3x-1) - (x-7) \equiv 5(x+1)$

a) P only (Q and R are equations)

$$b) 6x - 2 - x + 7 = 5x + 5 = 5(x+1)$$

2 Identities: finding unknowns

a) $4(2x+a) \equiv 8x - 20$. Find a .

b) $(x+a)(x+5) \equiv x^2 + bx - 15$. Find a and b .

$$a) 4a = -20, a = -5$$

$$b) 5a = -15, \text{ so } a = -3$$

$$b = a + 5 = 2$$

3 Simplifying algebraic fractions

Simplify fully

$$a) \frac{12x^3y}{18xy^4}$$

$$b) \frac{x^2-16}{2x^2+7x-4}$$

$$a) \frac{2x^2}{3y^3}$$

$$b) \frac{(x-4)(x+4)}{(2x-1)(x+4)} = \frac{x-4}{2x-1}$$

4 Multiplying and dividing

Simplify fully

$$a) \frac{x^2+3x}{10} \times \frac{15}{x^2-9}$$

$$b) \frac{4x}{x-1} \div \frac{x^2}{3x-3}$$

$$a) \frac{x(x+3)}{10} \times \frac{15}{(x-3)(x+3)} = \frac{3x}{2(x-3)}$$

$$b) \frac{4x}{x-1} \times \frac{3(x-1)}{x^2} = \frac{12}{x}$$

Algebraic fractions and algebraic proof revision mat answers

Higher A

5 Adding and subtracting

Write as a single fraction in its simplest form

$$a) \frac{2x-1}{4} - \frac{x+3}{6}$$

$$b) \frac{5}{x-2} - \frac{3}{x+1}$$

$$a) \frac{3(2x-1) - 2(x+3)}{12} = \frac{4x-9}{12}$$

$$b) \frac{5(x+1) - 3(x-2)}{(x-2)(x+1)} = \frac{2x+11}{(x-2)(x+1)}$$

6 Algebraic fractions: equations

Solve $\frac{2}{x} + \frac{3}{x+2} = 1$

$$2(x+2) + 3x = x(x+2)$$

$$5x + 4 = x^2 + 2x$$

$$x^2 - 3x - 4 = 0, (x-4)(x+1) = 0$$

$$x = 4 \text{ or } x = -1$$

7 Algebraic proof

a) Prove that the sum of any three consecutive even numbers is a multiple of 6.

b) Prove that $(3n+1)^2 - (3n-1)^2$ is a multiple of 12 for any whole number n .

$$a) 2n + (2n+2) + (2n+4) = 6n + 6 = 6(n+1)$$

$$b) 9n^2 + 6n + 1 - (9n^2 - 6n + 1)$$

$$= 12n, \text{ a multiple of 12}$$

8 Algebraic proof: true or false?

a) Ana says $n^2 + n + 11$ is a prime number for every whole number n . Show that she is wrong.

b) Prove that the product of two consecutive whole numbers, added to the larger number, is always a square number.

$$a) n = 10: 121 = 11 \times 11, \text{ not prime}$$

$$b) n(n+1) + (n+1)$$

$$= (n+1)(n+1) = (n+1)^2$$