

|                                                                                                                                                  |                                                                                                                                                                           |                                                                                                                                                                                   |                                                                                                                                                               |
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| <b>a</b> Show that<br>$(x+5)(x-1) - (x+2)^2 \equiv -9$<br><br>$x^2 + 4x - 5 - x^2 - 4x - 4 = -9$<br>an identity that leaves a number             | <b>b</b> $n$ is an integer.<br>Write $2n+1$ plus the next odd number as a multiple of 4.<br><br>$4(n+1)$<br>the next odd number, then a multiple                          | <b>c</b> Explain why<br>$2n+7$ is odd for every integer $n$ .<br><br>$2n+7 = 2(n+3) + 1$<br>explaining odd from its form                                                          | <b>d</b> $x^2 > x$ for every value of $x$ .<br>Give a counterexample.<br><br>$x = \frac{1}{2}$ (any $0 \leq x \leq 1$ )<br>a counterexample that is not whole |
| <b>e</b> Prove that the sum of four consecutive integers is never a multiple of 4.<br><br>$4n+6 = 4(n+1) + 2$<br>proving something is never true | <b>f</b> Prove that the product of any two odd numbers is odd.<br><br>$(2m+1)(2n+1) = 2(2mn+m+n)+1$<br>two different letters for two numbers                              | <b>g</b> Find $a$ and $b$ so that<br>$(x+a)^2 - (x-b)^2 \equiv 10x+5$<br><br>$a=3, b=2$<br>two equations from one identity                                                        | <b>h</b> Prove that $x^2 - 6x + 10$ is positive for every value of $x$ .<br><br>$(x-3)^2 + 1 \geq 1$<br>always positive: complete the square                  |
| <b>i</b> Prove that $3^{n+2} - 3^n$ is a multiple of 8 for every positive integer $n$ .<br><br>$3^n(9-1) = 8 \times 3^n$<br>indices in a proof   | <b>j</b> Prove that $n^3 - n$ is a multiple of 6 for every integer $n$ .<br><br>$(n-1)n(n+1)$<br><i>three consecutive integers</i><br>factorise into consecutive integers | <b>k</b> Prove that the square of any odd number is one more than a multiple of 8.<br><br>$4n(n+1)+1$<br><i>and <math>n(n+1)</math> is even</i><br>an even product inside a proof | <b>l</b> Prove that $n(n+1)(n+2)(n+3)+1$ is a square number.<br><br>$(n^2+3n+1)^2$<br>pairing brackets to find a square                                       |

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