Iteration

ANSWERS

1

Start with $x_0 = 6$. Find x_1, x_2 and x_3.

$$x_{n+1} = \frac{x_n + 14}{2}$$

x_0	x_1	x_2	x_3
6	10	12	13

$x_1 = \frac{6+14}{2} = 10$

$x_2 = \frac{10+14}{2} = 12$

$x_3 = \frac{12+14}{2} = 13$

2

Start with $x_0 = 3$. Find x_1, x_2 and x_3.

$$x_{n+1} = 3x_n - 4$$

x_0	x_1	x_2	x_3
3	5	11	29

$x_1 = 3 \times 3 - 4 = 5$

$x_2 = 3 \times 5 - 4 = 11$

$x_3 = 3 \times 11 - 4 = 29$

3

Start with $x_0 = 1$. Find x_1, x_2 and x_3.

$$x_{n+1} = \frac{x_n + 17}{2}$$

x_0	x_1	x_2	x_3
1	9	13	15

$x_1 = \frac{1+17}{2} = 9$

$x_2 = \frac{9+17}{2} = 13$

$x_3 = \frac{13+17}{2} = 15$

4 *Notice the change!*

Start with $x_0 = 3$. Find x_1, x_2 and x_3 to 3 decimal places.

$$x_{n+1} = \sqrt{x_n + 5}$$

x_0	x_1	x_2	x_3
3	2.828	2.798	2.792

$x_1 = \sqrt{3+5} = 2.828$

$x_2 = \sqrt{2.828+5} = 2.798$

$x_3 = \sqrt{2.798+5} = 2.792$

5

Start with $x_0 = 1$. Find x_1, x_2 and x_3 to 3 decimal places.

$$x_{n+1} = \frac{x_n + 7}{3}$$

x_0	x_1	x_2	x_3
1	2.667	3.222	3.407

$x_1 = \frac{1+7}{3} = 2.667$

$x_2 = \frac{2.667+7}{3} = 3.222$

$x_3 = \frac{3.222+7}{3} = 3.407$

6

Ali uses this formula with $x_0 = 10$.

He writes $x_1 = \sqrt{10} + 6 = 9.162$.

Explain his mistake. Then find x_1 and x_2.

$$x_{n+1} = \sqrt{x_n + 6}$$

He square rooted 10, not 10 + 6

$x_1 = \sqrt{10+6} = \sqrt{16} = 4$

$x_2 = \sqrt{4+6} = 3.162$

7 *Notice the change!*

Start with $x_0 = 2$. Keep going until two values in a row agree to 2 decimal places.

$$x_{n+1} = \sqrt{x_n + 7}$$

	4 d.p.	2 d.p.
x_1	3	3.00
x_2	3.1623	3.16
x_3	3.1878	3.19
x_4	3.1918	3.19
x_5		
x_6		

Solution 3.19 (2 d.p.)

8

Start with $x_0 = 3$. Keep going until two values in a row agree to 2 decimal places.

$$x_{n+1} = \sqrt{2x_n + 5}$$

	4 d.p.	2 d.p.
x_1	3.3166	3.32
x_2	3.4108	3.41
x_3	3.4382	3.44
x_4	3.4462	3.45
x_5	3.4485	3.45
x_6		

Solution 3.45 (2 d.p.)

Iteration *further practice*

ANSWERS

9

Start with $x_0 = 0$. Find x_3.

$$x_{n+1} = \frac{x_n + 24}{2}$$

$x_1 = 12$, $x_2 = 18$

Answer $x_3 = 21$

10

Start with $x_0 = 1$. Find x_1, x_2 and x_3 to 3 decimal places.

$$x_{n+1} = \sqrt{x_n + 20}$$

$x_1 = \sqrt{21} = 4.583$

Answer 4.583, 4.958, 4.996

11

Jo saves. A_n is the amount in her account, in £, at the start of year n, and $A_0 = 1000$. How much is in the account at the start of year 3?

$$A_{n+1} = 1.05A_n + 100$$

$A_1 = 1150$, $A_2 = 1307.50$

$A_3 = 1472.875$

Answer £1472.88

12

Start with $x_0 = 3$. Keep going until two values in a row agree to 2 decimal places.

$$x_{n+1} = \sqrt{x_n + 13}$$

4, 4.1231

4.1380, 4.1398

Answer 4.14 (2 d.p.)

13

$x_1 = 4$. Find x_0.

$$x_{n+1} = \frac{x_n + 10}{4}$$

$\frac{x_0+10}{4} = 4$

$x_0 + 10 = 16$

Answer $x_0 = 6$

14

A cup of tea cools. T_n is its temperature in °C after n minutes, and $T_0 = 80$.

After how many minutes is it first below 60 °C?

$$T_{n+1} = 0.9T_n + 2$$

74, 68.6, 63.74

$T_4 = 59.366$

Answer 4 minutes

15

Start with $x_0 = 3$. Keep going until two values in a row agree to 2 decimal places.

$$x_{n+1} = 3 + \frac{2}{x_n}$$

3.6667, 3.5455

3.5641, 3.5612

Answer 3.56 (2 d.p.)

16

Mia starts with $x_0 = 5$. She says the values will never change. Is she right? Explain why.

$$x_{n+1} = \frac{x_n + 15}{4}$$

$\frac{5+15}{4} = 5$, so $x_1 = 5$

every turn puts 5 in and gets 5 out

Answer Yes