

1



Use $x_{n+1} = \sqrt[3]{6x_n - 3}$ with $x_0 = 2$ to find a solution of $x^3 - 6x + 3 = 0$ to 3 decimal places. Use a sign change to show your answer is correct to 3 decimal places.

$$2.0801, \dots, 2.1447, 2.1449$$

$$2.1445: -0.0047, 2.1455: 0.0031$$

$$x = 2.145$$

Answer

2



A daily medicine dose, in mg, follows $D_{n+1} = 0.6D_n + 80$ with $D_0 = 80$. Find the amount it settles at, and explain using algebra why the amount gets closer to it every day.

$$D = 0.6D + 80, \text{ so } 0.4D = 80$$

$$D_{n+1} - 200 = 0.6(D_n - 200)$$

the gap to 200 is $\times 0.6$ each day

$$200 \text{ mg}$$

Answer

3



A cone has height x cm, base radius $(x + 1)$ cm and volume 50 cm^3 .

Show that

$$x = \sqrt{\frac{150}{\pi x}} - 1$$

then use it with $x_0 = 3$ to find x to 2 decimal places.

$$\frac{1}{3}\pi(x+1)^2x = 50, (x+1)^2 = \frac{150}{\pi x}$$

$$2.9894 \dots, 2.9918, 2.9949$$

$$x = 2.99$$

Answer

4



Use

$$x_{n+1} = \frac{x_n + \frac{20}{x_n}}{2}$$

with $x_0 = 4$ to find x_3 to 6 decimal places. Show that, if the values settle at L , then $L^2 = 20$.

$$x_1 = 4.5000, x_2 = 4.472222$$

$$2L = L + \frac{20}{L}, \text{ so } L^2 = 20$$

$$x_3 = 4.472136$$

Answer

5



$x_{n+1} = 1 + \frac{1}{x_n}$ with $x_0 = 1$ gives values that settle at a number α . Find α exactly.

$$\alpha = 1 + \frac{1}{\alpha}, \text{ so } \alpha^2 - \alpha - 1 = 0$$

$$\alpha = \frac{1 \pm \sqrt{5}}{2}, \text{ positive as every value is}$$

$$\alpha = \frac{1 + \sqrt{5}}{2}$$

Answer

6



$x_{n+1} = \frac{x_n^2 + 6}{5}$ with $x_0 = 0$ gives values that settle. Find exactly what they settle at. Explain why it is not the other solution of $L = \frac{L^2 + 6}{5}$.

$$L^2 - 5L + 6 = 0, L = 2 \text{ or } 3$$

$$\text{if } x_n < 2, x_{n+1} < \frac{4+6}{5} = 2$$

so the values never pass 2

$$2$$

Answer

7



$x^2 - 5x + 3 = 0$ can be rearranged as

$$A: x_{n+1} = \frac{x_n^2 + 3}{5} \quad \text{or} \quad B: x_{n+1} = 5 - \frac{3}{x_n}$$

Starting from $x_0 = 4$, which one finds the larger solution? Find it to 2 decimal places.

$$A: 3.80, 3.49, 3.03 \dots \text{ falls to } 0.70$$

$$B: 4.2500 \dots, 4.3014, 4.3025$$

$$B, x = 4.30$$

Answer

8



Use $x_{n+1} = \frac{1}{1 + x_n^2}$ with $x_0 = 1$ to solve $x^3 + x - 1 = 0$ to 2 decimal places. Explain why the solution always lies between two values in a row.

$$0.500, 0.800, \dots, 0.680, 0.684$$

a bigger x_n gives a smaller x_{n+1}

so the values go above, below, above

$$x = 0.68$$

Answer