

# Iteration

1 each answer is the next input

Start with  $x_0 = 4$ . Find  $x_1, x_2$  and  $x_3$ . Round to 3 decimal places if you need to.

$$x_{n+1} = \frac{x_n + 8}{4}$$

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3$   
 $4 \rightarrow 3 \rightarrow 2.75 \rightarrow 2.688$

$x_1 = \frac{4 + 8}{4}$   
 $x_2 = \frac{3 + 8}{4}$   
 $x_3 = \frac{2.75 + 8}{4}$

2

Start with  $x_0 = 1$ . Find  $x_1, x_2$  and  $x_3$ . Round to 3 decimal places if you need to.

$$x_{n+1} = \frac{x_n + 10}{5}$$

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3$   
 $1 \rightarrow 2.2 \rightarrow 2.44 \rightarrow 2.488$

$x_1 = \frac{1 + 10}{5}$   
 $x_2 = \frac{2.2 + 10}{5}$   
 $x_3 = \frac{2.44 + 10}{5}$

3

Start with  $x_0 = 1$ . Find  $x_1, x_2$  and  $x_3$ . Round to 3 decimal places if you need to.

$$x_{n+1} = \frac{2x_n + 3}{4}$$

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3$   
 $1 \rightarrow 1.25 \rightarrow 1.375 \rightarrow 1.438$

$x_1 = \frac{2 \times 1 + 3}{4}$   
 $x_2 = \frac{2 \times 1.25 + 3}{4}$   
 $x_3 = \frac{2 \times 1.375 + 3}{4}$

4

Start with  $x_0 = 3$ . Find  $x_1, x_2$  and  $x_3$ . Round to 3 decimal places if you need to.

$$x_{n+1} = \frac{x_n + 12}{3}$$

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3$   
 $3 \rightarrow 5 \rightarrow 5.667 \rightarrow 5.889$

$x_1 = \frac{3 + 12}{3}$   
 $x_2 = \frac{5 + 12}{3}$   
 $x_3 = \frac{5.667 + 12}{3}$

5 *Notice the change!*

Start with  $x_0 = 2$ . Find  $x_1, x_2$  and  $x_3$ . Round to 3 decimal places if you need to.

$$x_{n+1} = \sqrt{x_n + 5}$$

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3$   
 $2 \rightarrow 2.646 \rightarrow 2.765 \rightarrow 2.787$

$x_1 = \sqrt{2 + 5}$   
 $x_2 = \sqrt{2.646 + 5}$   
 $x_3 = \sqrt{2.765 + 5}$

6

Start with  $x_0 = 3$ . Find  $x_1, x_2$  and  $x_3$ . Round to 3 decimal places if you need to.

$$x_{n+1} = \sqrt{2x_n + 7}$$

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3$   
 $3 \rightarrow 3.606 \rightarrow 3.77 \rightarrow 3.813$

$x_1 = \sqrt{2 \times 3 + 7}$   
 $x_2 = \sqrt{2 \times 3.606 + 7}$   
 $x_3 = \sqrt{2 \times 3.77 + 7}$

7

*Notice the change!*

Start with  $x_0 = 2$ . Keep going until two values in a row agree to 2 decimal places.

$$x_{n+1} = 3\sqrt{x_n + 10}$$

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow$   
 $2 \rightarrow 2.2894 \rightarrow 2.3077 \rightarrow$   
 $x_3 \rightarrow x_4 \rightarrow x_5$   
 $2.3088 \rightarrow \rightarrow \rightarrow$

Solution 2.31 (2 d.p.)

8

Start with  $x_0 = 2$ . Keep going until two values in a row agree to 2 decimal places.

$$x_{n+1} = 3\sqrt{3x_n + 7}$$

$x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow$   
 $2 \rightarrow 2.3513 \rightarrow 2.4132 \rightarrow$   
 $x_3 \rightarrow x_4 \rightarrow x_5$   
 $2.4238 \rightarrow 2.4256 \rightarrow 2.4259$

Solution 2.43 (2 d.p.)

## Iteration further practice

9

Use  $x_{n+1} = \sqrt{2x_n + 9}$  with  $x_0 = 2$  to find a solution to 2 decimal places. Keep going until two values in a row agree.

2.3513, 2.3930

2.3978, 2.3984

Answer 2.40

10

Start with  $x_0 = 3$ . Use  $x_{n+1} = 3 - \frac{1}{x_n}$  to find  $x_1, x_2$  and  $x_3$ . Round to 3 decimal places.

$3 - \frac{1}{3} = \frac{8}{3}$ , then  $3 - \frac{3}{8}$

Answer 2.667, 2.625, 2.619

11

Show that  $x^3 - 6x + 2 = 0$  has a solution between  $x = 2$  and  $x = 3$ .

$x = 2: 8 - 12 + 2 = -2$

$x = 3: 27 - 18 + 2 = 11$

Answer it changes sign, so a solution

12

The formula  $x_{n+1} = \sqrt{5 - x_n}$  gives a solution of a quadratic equation. Find the equation in the form  $x^2 + bx + c = 0$ .

$x = \sqrt{5 - x}$ , so  $x^2 = 5 - x$

Answer  $x^2 + x - 5 = 0$

13

Each year Maya's savings grow by 4% and then she adds £50, so  $S_{n+1} = 1.04S_n + 50$ . She starts with  $S_0 = £1000$ . How much does she have after 3 years?

£1090.00, £1183.60, £1280.94

Answer £1280.94

14

A box has a square base of side  $x$  cm and a height of  $(x + 2)$  cm. Its volume is  $30 \text{ cm}^3$ .

Use  $x_{n+1} = \sqrt{\frac{30}{x_n + 2}}$  with  $x_0 = 3$  to find  $x$  to 2 decimal places.

2.4495, 2.5966, 2.5547

2.5664, 2.5631, 2.5641

Answer 2.56 cm

15

A rectangle is  $x$  cm wide and  $(x + 3)$  cm long. Its area is  $20 \text{ cm}^2$ . Show that  $x = \sqrt{20 - 3x}$ .

$x(x + 3) = 20$ , so  $x^2 + 3x = 20$

$x^2 = 20 - 3x$ , so  $x = \sqrt{20 - 3x}$

Answer shown

16

Ana's iteration gives a solution of  $x^3 - 6x + 2 = 0$  as 2.26 to 2 decimal places. Show that she is right.

$x = 2.255: -0.063$ , negative

$x = 2.265: 0.030$ , positive

Answer sign change, so 2.26 is right