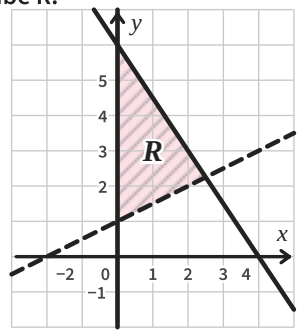
Inequalities and regions

ANSWERS

1

Write down the three inequalities that describe R.



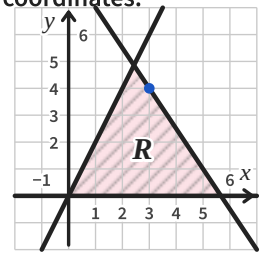
gradients 1/2 and −3/2

(1, 3) is in R

Answer x ≥ 0, 2y > x + 2, 3x + 2y ≤ 12

2

R is the region where $y \geq 0$, $y \leq 2x$ and $3x + 2y \leq 17$. Find the greatest value of $x + 2y$ at a point in R with integer coordinates.



corner (17/7, 34/7): not whole

(2, 5) is not in R; 3 + 8 = 11

Answer 11, at (3, 4)

3

R is the region where $0 \leq x \leq 3$, $y \geq 2x$ and $y < x + k$, where k is a whole number. R contains exactly 14 points with integer coordinates. Find k.

x = 0, 1, 2, 3: k, k − 1, k − 2, k − 3 points

4k − 6 = 14, k = 5

Answer k = 5

4

R is the region where $y \geq 0$, $y \leq 2x + 1$ and $x + 3y \leq 9$. Find the exact coordinates of the highest point of R.

$x + 3(2x + 1) = 9$, $7x = 6$

$x = \frac{6}{7}$, $y = \frac{19}{7}$

Point $(\frac{6}{7}, \frac{19}{7})$

5

R is the set of points closer to the y axis than to the line $x = 6$, below the line $y = x + 2$ and above the x axis.
Write down three inequalities for R.
How many points with integer coordinates are in R?

x < 3, y < x + 2, y > 0

(0, 1), (1, 1), (1, 2),

(2, 1), (2, 2), (2, 3)

Answer 6 points

6

R is the region where $y > 2x$ and $y < 2x + 1$. Show that R has points in it, but no point with integer coordinates.

(0, 0.5) is in R: 0 < 0.5 < 1

x whole: no whole y between 2x and 2x + 1

Reason no integer points

7

R is the region where $y \geq 1$, $y \leq 2x + 3$ and $y \leq k - 2x$. R is a triangle with an area of 8 square units. Find k.

corners (−1, 1), ((k − 1)/2, 1)

base = height = (k + 1)/2; $(k + 1)^2 = 64$

Answer k = 7

8

A hall hires small tables that seat 4 for £8 each and large tables that seat 6 for £14 each. It needs seats for at least 72 people, has room for 15 tables at most and wants at least as many small tables as large.
Find the cheapest order.

4x + 6y ≥ 72, x + y ≤ 15, x ≥ y

(9, 6): 72 seats, 15 tables, £72 + £84

Answer 9 small, 6 large: £156