

Graph Transformations: Reflections *extend*

ANSWERS

1



$$f(x) = x^3 - 3x^2 + 2.$$

Show that $-f(-x) = x^3 + 3x^2 - 2$.

$$f(-x) = -x^3 - 3x^2 + 2$$

$$-f(-x) = x^3 + 3x^2 - 2$$

$$-f(-x) = x^3 + 3x^2 - 2$$

Show

2



$y = f(x)$ is a quadratic graph with turning point $(3, -2)$ that passes through $(1, 6)$. Find $y = f(-x)$ in the form $y = ax^2 + bx + c$.

$$y = a(x - 3)^2 - 2, 6 = 4a - 2, a = 2$$

$$2(-x - 3)^2 - 2 = 2(x + 3)^2 - 2$$

$$y = 2x^2 + 12x + 16$$

Answer

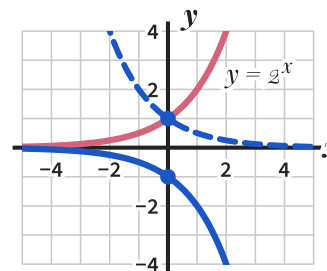
3



Sketch $y = 2^{-x}$ and $y = -2^x$.

Where does each cross the y axis?

Explain why they never meet.



one is always above the x axis, the other always below

$$(0, 1) \text{ and } (0, -1)$$

Answer

4



$f(x) = x^3 + x^2 - 9x$. Find the three points where the graph of $y = f(x)$ meets its reflection in the y axis.

$$x^3 + x^2 - 9x = -x^3 + x^2 + 9x$$

$$2x^3 - 18x = 0, x = 0 \text{ or } \pm 3$$

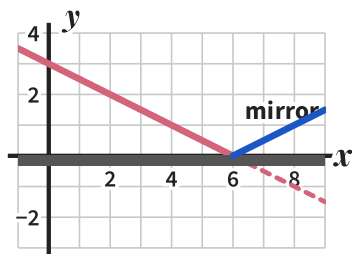
$$(-3, 9), (0, 0), (3, 9)$$

Answer

5



A laser beam along $y = 3 - \frac{x}{2}$ hits a mirror on the x axis. The reflected beam is the reflection of the dotted line in the x axis. Find its equation.



$$y = \frac{x}{2} - 3$$

Answer

6



$f(x) = x^2 - 2x - 3$. Find the exact coordinates of the points where the graph of $y = f(x)$ meets the graph of $y = -f(-x)$.

$$-f(-x) = -x^2 - 2x + 3$$

$$x^2 - 2x - 3 = -x^2 - 2x + 3: x^2 = 3$$

$$(\sqrt{3}, -2\sqrt{3}), (-\sqrt{3}, 2\sqrt{3})$$

Answer

7



Use reflections of the graphs to explain why $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$ for every angle x .

$\sin$: a y axis mirror = an x axis mirror

$\cos$: the y axis is a line of symmetry

$\sin$ turns, $\cos$ is symmetrical

Reason

8



$f(x) = ax^2 + bx + c$. Prove that the graphs of $y = -f(x)$ and $y = f(-x)$ are the same only when $a = 0$ and $c = 0$. What kind of graph is $y = f(x)$ then?

$$-ax^2 - bx - c = ax^2 - bx + c$$

$$2ax^2 + 2c = 0 \text{ for every } x$$

A straight line through $(0, 0)$

Proof