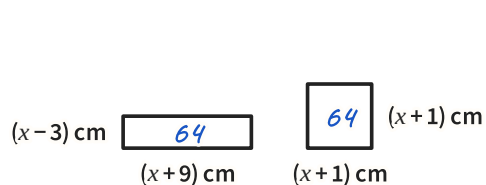


Geometry Puzzles with Algebra

ANSWERS

1

The rectangle and the square have the same area. Find the value of x .

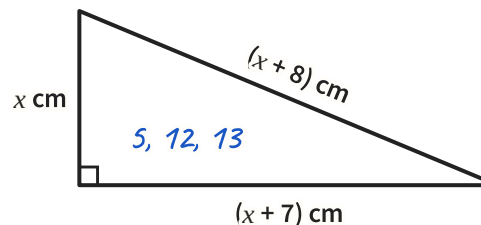


$$\begin{aligned} &(x+9)(x-3) \text{ and } (x+1)^2 \\ &(x+9)(x-3) = (x+1)^2 \\ &x^2 + 6x - 27 = x^2 + 2x + 1 \\ &4x = 28, \text{ so } x = 7 \end{aligned}$$

$$x = \underline{7}$$

2

Find the perimeter of the triangle.

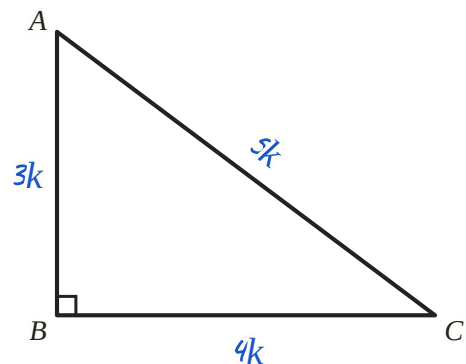


$$\begin{aligned} &\text{sides } x, x+7, x+8 \\ &x^2 + (x+7)^2 = (x+8)^2 \\ &x^2 - 2x - 15 = 0 \\ &(x-5)(x+3) = 0, \text{ so } x = 5 \\ &\text{Perimeter} = 30 \text{ cm} \end{aligned}$$

$$\text{Perimeter} = \underline{30 \text{ cm}}$$

3

Triangle ABC has a right angle at B . $AB : BC = 3 : 4$. The area of the triangle is 96 cm^2 . Find its perimeter.

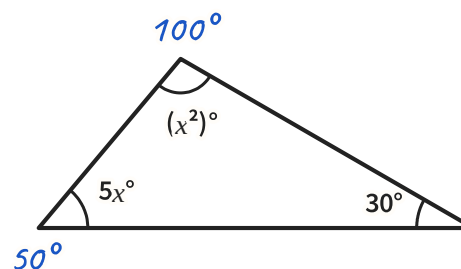


$$\begin{aligned} &AB = 3k, BC = 4k \\ &\frac{1}{2} \times 3k \times 4k = 96, \text{ so } 6k^2 = 96 \\ &k^2 = 16, \text{ so } k = 4: AB = 12, BC = 16 \\ &AC = \sqrt{12^2 + 16^2} = 20, \\ &\text{perimeter} = 48 \text{ cm} \end{aligned}$$

$$\text{Perimeter} = \underline{48 \text{ cm}}$$

4

Find the size of the largest angle in the triangle.



$$\begin{aligned} &x^2, 5x, 30 \\ &x^2 + 5x + 30 = 180 \\ &x^2 + 5x - 150 = 0, \text{ so } \\ &(x+15)(x-10) = 0 \\ &\text{Largest angle} = 100^\circ \end{aligned}$$

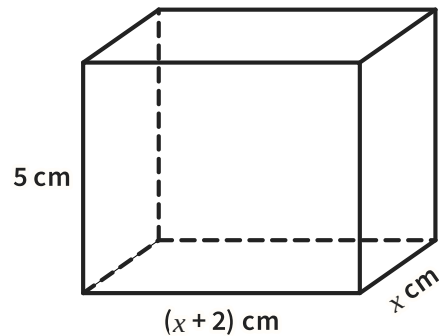
$$\text{Largest angle} = \underline{100^\circ}$$

Geometry Puzzles with Algebra

ANSWERS

5

The volume of the cuboid is 120 cm^3 . Find its surface area.

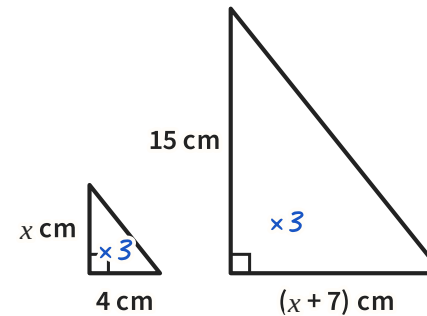


$$\begin{aligned} 5x(x+2) \\ 5x(x+2) &= 120 \\ x^2 + 2x - 24 &= 0, \text{ so} \\ (x+6)(x-4) &= 0, x = 4 \\ 2(24 + 30 + 20) &= 148 \text{ cm}^2 \end{aligned}$$

Surface area = 148 cm²

6

The two triangles are similar. Find the value of x .

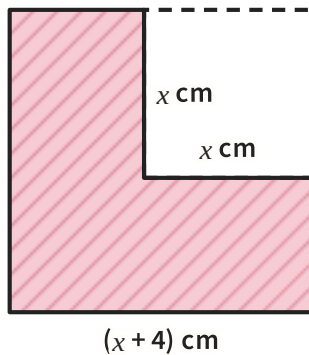


$$\begin{aligned} x \leftrightarrow 15, 4 \leftrightarrow x+7 \\ \frac{x}{15} &= \frac{4}{x+7} \\ x^2 + 7x &= 60, \text{ so } x^2 + 7x - 60 = 0 \\ (x+12)(x-5) &= 0, \text{ so } x = 5 \end{aligned}$$

$x =$ 5

7

A square of side $x \text{ cm}$ is cut from the corner of a square of side $(x+4) \text{ cm}$. The shaded shape has an area of 56 cm^2 . Find its perimeter.

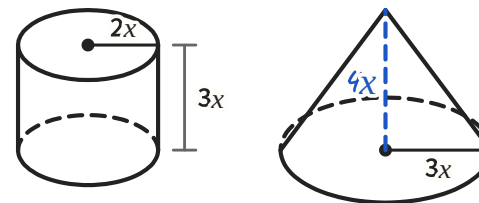


$$\begin{aligned} (x+4)^2 - x^2 \\ (x+4)^2 - x^2 &= 56 \\ 8x + 16 &= 56, \text{ so } x = 5 \\ 4 \times 9 &= 36 \text{ cm} \end{aligned}$$

Perimeter = 36 cm

8

The cylinder has radius $2x$ and height $3x$. The cone has radius $3x$. They have the same volume. Find the height of the cone in terms of x . The volume of a cone is $\frac{1}{3}\pi r^2 h$.



$$\begin{aligned} \text{cylinder } 12\pi x^3, \text{ cone } 3\pi x^2 h \\ 3\pi x^2 h &= 12\pi x^3 \\ h &= 4x \end{aligned}$$

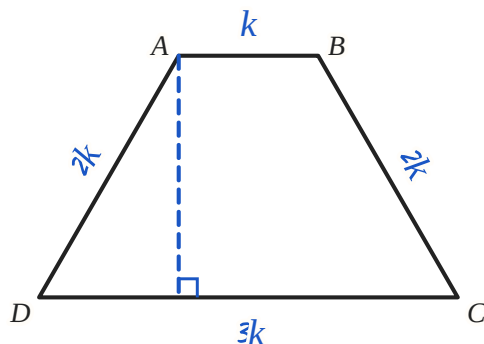
Height = 4x

Geometry Puzzles with Algebra *further practice*

ANSWERS

9

$ABCD$ is an isosceles trapezium with $AB : BC : CD = 1 : 2 : 3$ and a perimeter of 48 cm. Find its area as a surd.

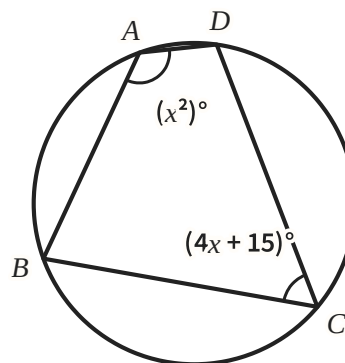


$$\begin{aligned} AB &= k, BC = AD = 2k, CD = 3k \\ 8k &= 48, \text{ so } k = 6 \\ h^2 &= 12^2 - 6^2 = 108, \text{ so } h = 6\sqrt{3} \\ \frac{1}{2} \times (6 + 18) \times 6\sqrt{3} &= 72\sqrt{3} \text{ cm}^2 \end{aligned}$$

Area = $72\sqrt{3} \text{ cm}^2$

10

$ABCD$ is a cyclic quadrilateral. Find the size of angle A.

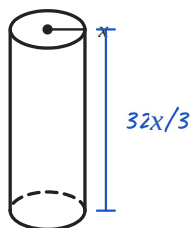
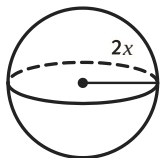


$$\begin{aligned} A &= x^2, C = 4x + 15 \\ x^2 + 4x + 15 &= 180 \\ x^2 + 4x - 165 &= 0, \text{ so } \\ (x + 15)(x - 11) &= 0 \\ \text{Angle } A &= 11^2 = 121^\circ \end{aligned}$$

Angle A = 121°

11

A solid sphere of radius $2x$ is melted down and made into a cylinder of radius x . None is lost. Find the height of the cylinder in terms of x . The volume of a sphere is $\frac{4}{3}\pi r^3$.

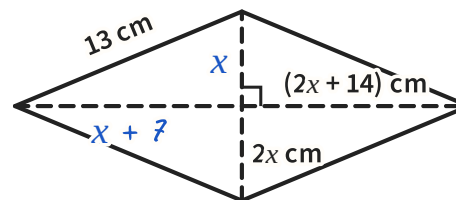


$$\begin{aligned} \frac{4}{3}\pi(2x)^3 &= \frac{32}{3}\pi x^3 \\ \pi x^2 h &= \frac{32}{3}\pi x^3 \\ h &= \frac{32x}{3} \end{aligned}$$

Height = $\frac{32x}{3}$

12

The rhombus has sides of 13 cm. Its diagonals are $2x$ cm and $(2x + 14)$ cm long. Find its area.

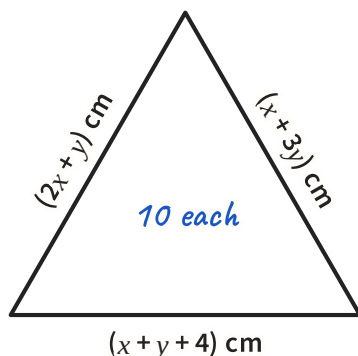


$$\begin{aligned} \text{halves } x \text{ and } x + 7 \\ x^2 + (x + 7)^2 &= 13^2 \\ x^2 + 7x - 60 &= 0, \text{ so } \\ (x + 12)(x - 5) &= 0, x = 5 \\ \text{Area} &= \frac{1}{2} \times 10 \times 24 = 120 \text{ cm}^2 \end{aligned}$$

Area = 120 cm^2

13

The triangle is equilateral. Find its area. Give your answer as a surd.

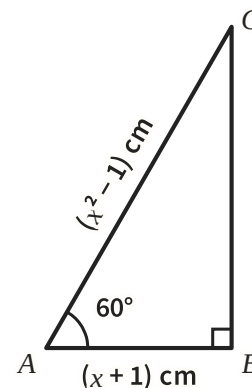


$$\begin{aligned} 2x+y &= x+3y = x+y+4 \\ x &= 2y \\ x &= 4, \text{ so } y = 2: \text{ each side is } 10 \\ \frac{1}{2} \times 10 \times 10 \times \sin 60^\circ &= 25\sqrt{3} \text{ cm}^2 \end{aligned}$$

Area = $25\sqrt{3} \text{ cm}^2$

14

Find the length of BC . Give your answer as a surd.



$$\cos 60^\circ = \frac{x+1}{x^2-1}$$

$$\frac{x+1}{x^2-1} = \frac{1}{2}, \text{ so } x^2-1 = 2x+2$$

$$x^2-2x-3 = 0, \text{ so}$$

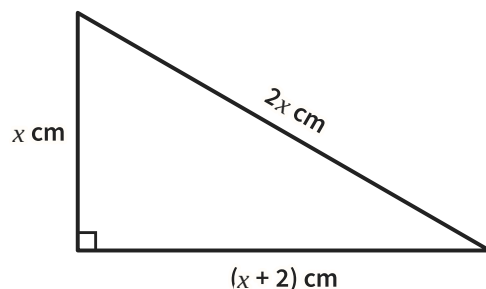
$$(x-3)(x+1) = 0, x = 3$$

$$BC = \sqrt{(8^2-4^2)} = \sqrt{48} = 4\sqrt{3} \text{ cm}$$

BC = $4\sqrt{3} \text{ cm}$

15

Find the perimeter of the triangle. Give your answer in the form $a + b\sqrt{3}$.

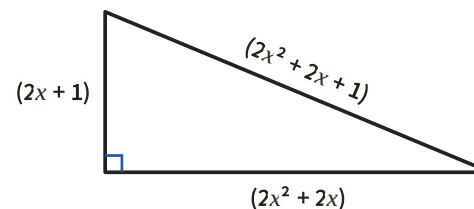


$$\begin{aligned} \text{sides } x, x+2, 2x \\ x^2 + (x+2)^2 &= (2x)^2 \\ x^2 - 2x - 2 &= 0 \\ x &= 1 + \sqrt{3} \\ \text{Perimeter} &= 6 + 4\sqrt{3} \text{ cm} \end{aligned}$$

Perimeter = $6 + 4\sqrt{3} \text{ cm}$

16

Show that the triangle has a right angle for every positive value of x .



$$\begin{aligned} \text{shorter sides } 2x+1 \text{ and } 2x^2+2x \\ (2x+1)^2 + (2x^2+2x)^2 \\ = 4x^2 + 8x^3 + 8x^2 + 4x + 1 \\ (2x^2+2x+1)^2 = 4x^4 + 8x^3 + 8x^2 + \end{aligned}$$

The two sides are equal