

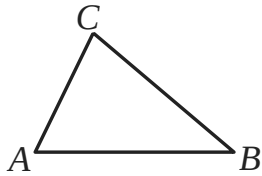
Fix it 1 Label the sides

Sine and cosine rules · ANSWERS

Each side takes the small letter of the angle opposite it: side a is opposite angle A . A side and its opposite angle make a pair.

EXAMPLE

Name the sides a , b and c .

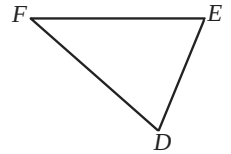


1. a is opposite angle A : BC .
2. b is opposite angle B : AC .
3. c is opposite angle C : AB .
4. Side a never touches corner A .

YOUR TURN

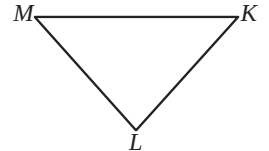
- 1 Which side is opposite angle E ? Name it with two capitals.

DF (side e)



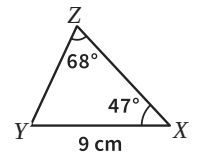
- 2 Which two corners does side m join?

K and L



- 3 Which marked angle makes a pair with the 9 cm side?

68° at Z



Check: Cover a corner with your finger. Its side is the one that does not touch your finger.

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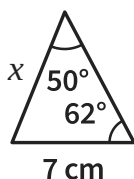
Fix it 2 Sine rule or cosine rule?

Sine and cosine rules · ANSWERS

Look for a pair: a side and its opposite angle, both known. A pair: the sine rule. No pair: the cosine rule. Two angles known? The third makes a pair.

EXAMPLE

Sine rule or cosine rule?

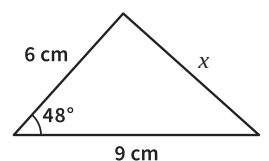


1. 7 cm is opposite 50°: a known pair.
2. x is opposite 62°.
3. A pair: the sine rule.

YOUR TURN

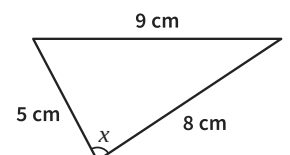
- 1 Sine rule or cosine rule?

Cosine rule: no pair



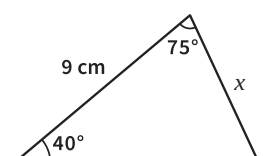
- 2 Sine rule or cosine rule?

Cosine rule: no angle known



- 3 Sine rule or cosine rule?

Sine rule: 65° pairs with 9 cm



Check: For the sine rule, point to the pair. If you cannot, use the cosine rule.

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Fix it 3 Substitute: the sine rule

Sine and cosine rules · ANSWERS

Each side goes with the sine of its opposite angle. A side wanted: sides over sines, $\frac{a}{\sin A} = \frac{b}{\sin B}$. An angle wanted: sines over sides.

EXAMPLE

Angle A is 44° , angle B is 69° and side a is 9 cm. Substitute the numbers into the sine rule to find side b .

1. Pair b with B and a with A.
2. A side is wanted: sides over sines.
3. $\frac{b}{\sin 69^\circ} = \frac{9}{\sin 44^\circ}$

YOUR TURN

- 1 Angle D is 38° , angle E is 81° and side d is 6 cm. Substitute the numbers into the sine rule to find side e .

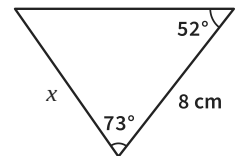
$$\frac{e}{\sin 81^\circ} = \frac{6}{\sin 38^\circ}$$

- 2 A side of 10 cm is opposite 66° . A side of 7 cm is opposite angle x . Substitute the numbers into the sine rule to find x .

$$\frac{\sin x}{7} = \frac{\sin 66^\circ}{10}$$

- 3 Substitute the numbers into the sine rule to find x .

$$\frac{x}{\sin 52^\circ} = \frac{8}{\sin 55^\circ}$$



Check: In each fraction, find the side and the angle on the triangle. They are opposite each other.

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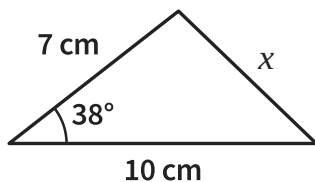
Fix it 4 Substitute: the cosine rule

Sine and cosine rules · ANSWERS

$a^2 = b^2 + c^2 - 2bc \cos A$. Side a is opposite the angle; b and c make the angle. An angle wanted: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

EXAMPLE

Substitute the numbers into the cosine rule.



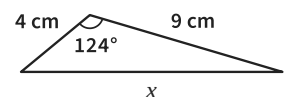
1. x is opposite 38° : it is a .
2. 7 and 10 make the 38° : b and c .
3. $x^2 = 7^2 + 10^2 - 2 \times 7 \times 10 \times \cos 38^\circ$

YOUR TURN

- 1 Sides 5 cm and 12 cm with 71° between them. Substitute the numbers into the cosine rule to find the third side, x .

$$x^2 = 5^2 + 12^2 - 2 \times 5 \times 12 \times \cos 71^\circ$$

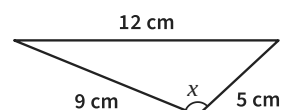
- 2 Substitute the numbers into the cosine rule.



$$x^2 = 4^2 + 9^2 - 2 \times 4 \times 9 \times \cos 124^\circ$$

- 3 Substitute the numbers into the cosine rule to find angle x .

$$\cos x = \frac{5^2 + 9^2 - 12^2}{2 \times 5 \times 9}$$



Check: The angle in cos is opposite the side on its own: a on the left, or a^2 taken away.

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Fix it 5 Solve for a side: the sine rule

Sine and cosine rules · ANSWERS

x as the numerator: multiply both sides by the sine under x . Then type it into the calculator in one go.

EXAMPLE

$\frac{x}{\sin 77^\circ} = \frac{5}{\sin 41^\circ}$. Find x to 1 decimal place.

1. x is the numerator: multiply both sides by $\sin 77^\circ$.
2. $x = \frac{5 \times \sin 77^\circ}{\sin 41^\circ}$
3. $x = 7.4$ cm

YOUR TURN

- 1 $\frac{x}{\sin 58^\circ} = \frac{11}{\sin 36^\circ}$. Find x to 1 decimal place.

$$x = 15.9 \text{ cm}$$

- 2 $\frac{x}{\sin 29^\circ} = \frac{14}{\sin 103^\circ}$. Find x to 1 decimal place.

$$x = 7.0 \text{ cm}$$

- 3 Angle F is 34° , angle G is 85° and side f is 4.6 cm. Find side g to 1 decimal place.

$$g = 8.2 \text{ cm}$$

Check: The bigger angle is opposite the longer side. Compare your side with the one you were given.

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Fix it 6 Solve for an angle: the sine rule

Sine and cosine rules · ANSWERS

Multiply both sides by the side under $\sin x$, so $\sin x$ is on its own. Then SHIFT and $\sin$: $x = \sin^{-1}(\quad)$.

EXAMPLE

$\frac{\sin x}{8} = \frac{\sin 63^\circ}{11}$. Find x to 1 decimal place.

1. Multiply both sides by 8.
2. $\sin x = \frac{8 \times \sin 63^\circ}{11} = 0.6480$
3. $x = \sin^{-1}(0.6480) = 40.4^\circ$

YOUR TURN

- 1 $\frac{\sin x}{5} = \frac{\sin 49^\circ}{8}$. Find x to 1 decimal place.

$$x = 28.1^\circ$$

- 2 $\frac{\sin x}{12} = \frac{\sin 71^\circ}{13}$. Find x to 1 decimal place.

$$x = 60.8^\circ$$

- 3 A side of 6.5 cm is opposite angle x . A side of 9.4 cm is opposite 112° . Find x to 1 decimal place.

$$x = 39.9^\circ$$

Check: $\sin x$ is never more than 1. If yours is, the fractions are upside down.

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Fix it 7 Solve for a side: the cosine rule

Sine and cosine rules · ANSWERS

BIDMAS: the squares, then $2 \times b \times c \times \cos A$. Take that away from $b^2 + c^2$. Then square root.

EXAMPLE

$x^2 = 9^2 + 6^2 - 2 \times 9 \times 6 \times \cos 41^\circ$. Find x to 1 decimal place.

1. $9^2 + 6^2 = 117$
2. $2 \times 9 \times 6 \times \cos 41^\circ = 81.51$
3. $x^2 = 117 - 81.51 = 35.49$
4. $x = \sqrt{35.49} = 6.0 \text{ cm}$

YOUR TURN

- 1 $x^2 = 6^2 + 11^2 - 2 \times 6 \times 11 \times \cos 43^\circ$. Find x to 1 decimal place.

$$x = \underline{7.8 \text{ cm}}$$

- 2 $x^2 = 3^2 + 8^2 - 2 \times 3 \times 8 \times \cos 131^\circ$. Find x to 1 decimal place.

$$x = \underline{10.2 \text{ cm}}$$

- 3 $x^2 = 8.5^2 + 6.2^2 - 2 \times 8.5 \times 6.2 \times \cos 27^\circ$. Find x to 1 decimal place.

$$x = \underline{4.1 \text{ cm}}$$

Check: Under 90° , x is shorter than Pythagoras gives. Over 90° , longer.

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Fix it 8 Solve for an angle: the cosine rule

Sine and cosine rules · ANSWERS

Work out the numerator and the denominator first, then divide to get $\cos x$. Then SHIFT and cos: $x = \cos^{-1}(\quad)$.

EXAMPLE

$\cos x = \frac{6^2 + 8^2 - 11^2}{2 \times 6 \times 8}$. Find x to 1 decimal place.

1. Numerator: $36 + 64 - 121 = -21$
2. Denominator: $2 \times 6 \times 8 = 96$
3. $\cos x = -21 \div 96 = -0.21875$
4. $x = \cos^{-1}(-0.21875) = 102.6^\circ$

YOUR TURN

- 1 $\cos x = \frac{7^2 + 9^2 - 5^2}{2 \times 7 \times 9}$. Find x to 1 decimal place.

$$x = \underline{33.6^\circ}$$

- 2 $\cos x = \frac{4^2 + 10^2 - 8^2}{2 \times 4 \times 10}$. Find x to 1 decimal place.

$$x = \underline{49.5^\circ}$$

- 3 A triangle has sides 3.5 cm, 6 cm and 8 cm. Find its largest angle to 1 decimal place.

$$\underline{112.0^\circ}$$

Check: $\cos x$ negative means x is obtuse. The largest angle is opposite the longest side.

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