

Fix it 1 First differences

Quadratic nth term · ANSWERS

Take each term away from the next one: the 2nd term – the 1st, the 3rd – the 2nd, and so on. A sequence that goes down has negative differences.

EXAMPLE

Find the first differences of
3, 7, 13, 21, 31, ...

1. $7 - 3 = 4$
2. $13 - 7 = 6$
3. $21 - 13 = 8$ and $31 - 21 = 10$
4. First differences: 4, 6, 8, 10

YOUR TURN

- 1 Find the first differences of
4, 12, 22, 34, 48, ...

8, 10, 12, 14

- 2 Find the first differences of
20, 17, 16, 17, 20, ...

-3, -1, 1, 3

- 3 Find the first differences of
-7, -4, 3, 14, 29, ...

3, 7, 11, 15

Check: Add each difference on to its term. You get the next term.

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Fix it 2 The second difference

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Find the first differences, then the differences between them. In a quadratic sequence this second difference is always the same.

EXAMPLE

Find the second difference of
2, 9, 20, 35, 54, ...

1. First differences: 7, 11, 15, 19
2. $11 - 7 = 4$, $15 - 11 = 4$, $19 - 15 = 4$
3. Second difference: 4

YOUR TURN

- 1 Find the second difference of
5, 7, 11, 17, 25, ...

2

- 2 Find the second difference of
12, 10, 14, 24, 40, ...

6

- 3 Find the second difference of
2, 5, 11, 20, 32, ...

3

Check: Work out every second difference along the row. They all match.

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Fix it 3 Halve it

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Halve the second difference. That is the number in front of n^2 : a second difference of 6 means the n th term starts $3n^2$.

EXAMPLE

The second difference is 10.
How does the n th term start?

1. Halve the second difference.
2. Half of 10 is 5.
3. The n th term starts $5n^2$.
4. Not $10n^2$: its second difference is 20.

YOUR TURN

- 1 The second difference is 8. How does the n th term start?

$$4n^2$$

- 2 The first differences are 5, 8, 11, 14. How does the n th term start?

$$\frac{3}{2}n^2$$

- 3 The second difference is -6 . How does the n th term start?

$$-3n^2$$

Check: Double the number in front of n^2 . You get the second difference back.

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Fix it 4 The n^2 part

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Substitute $n = 1, 2, 3, 4, 5$ into the n^2 part. Square n first, then multiply by the number in front (BIDMAS).

EXAMPLE

Work out the n^2 part, $4n^2$, for $n = 1$ to 5.

1. $n = 1$: $4 \times 1^2 = 4$
2. $n = 2$: $4 \times 2^2 = 4 \times 4 = 16$
3. $n = 3$: $4 \times 9 = 36$
4. n^2 part: 4, 16, 36, 64, 100

YOUR TURN

- 1 Work out $5n^2$ for $n = 1, 2, 3, 4$ and 5.

$$5, 20, 45, 80, 125$$

- 2 Work out $-2n^2$ for $n = 1, 2, 3, 4$ and 5.

$$-2, -8, -18, -32, -50$$

- 3 Work out $\frac{1}{2}n^2$ for $n = 1, 2, 3, 4$ and 5.

$$\frac{1}{2}, 2, 4\frac{1}{2}, 8, 12\frac{1}{2}$$

Check: Find the second difference of your n^2 part. It matches the sequence's.

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Fix it 5 What is left

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Take the n^2 part away from the sequence, term by term: each term take away the n^2 part under it.

EXAMPLE

Sequence 7, 14, 25, 40, 59
 n^2 part 2, 8, 18, 32, 50

1. Take the n^2 part away, term by term.
2. $7 - 2 = 5$, $14 - 8 = 6$, $25 - 18 = 7$
3. What is left: 5, 6, 7, 8, 9

YOUR TURN

- 1 Sequence 4, 13, 26, 43, 64
 n^2 part 2, 8, 18, 32, 50
 What is left?

2, 5, 8, 11, 14

- 2 Sequence 1, 8, 23, 46, 77
 n^2 part 4, 16, 36, 64, 100
 What is left?

-3, -8, -13, -18, -23

- 3 Sequence -3, 3, 19, 45, 81
 n^2 part 5, 20, 45, 80, 125
 What is left?

-8, -17, -26, -35, -44

Check: Add what is left on to the n^2 part. You get the sequence back.

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Fix it 6 The n th term of what is left

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What is left goes up (or down) by the same each time. Goes up by 3: start with $3n$, then add or take away to match. Goes down by 3: start with $-3n$. All the same number: it is that number.

EXAMPLE

What is left is 5, 6, 7, 8, 9.
 Find its n th term.

1. It goes up by 1, so start with n .
2. n : 1, 2, 3, 4, 5
3. Each one is 4 more: $n + 4$

YOUR TURN

- 1 What is left is 1, 4, 7, 10, 13. Find its n th term.

$3n - 2$

- 2 What is left is -2, -2, -2, -2, -2. Find its n th term.

-2

- 3 What is left is 8, 3, -2, -7, -12. Find its n th term.

$-5n + 13$ (or $13 - 5n$)

Check: Substitute $n = 1$ and $n = 2$. You get the first two numbers of what is left.

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Fix it 7 Put them together

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The n th term is the n^2 part plus the n th term of what is left. Write the n^2 term first, then the n term, then the number, each with its sign.

EXAMPLE

The n^2 part is $4n^2$. What is left has n th term $3 - 5n$.

1. n^2 part, then what is left.
2. $4n^2 + 3 - 5n$
3. n^2 term first, then n , then the number: $4n^2 - 5n + 3$

YOUR TURN

- 1 The n^2 part is $5n^2$. What is left has n th term $2n + 1$. Write the n th term.

$$5n^2 + 2n + 1$$

- 2 The n^2 part is $2n^2$. What is left has n th term $7 - 3n$. Write the n th term.

$$2n^2 - 3n + 7$$

- 3 The n^2 part is $\frac{1}{2}n^2$. What is left has n th term $4 - \frac{3}{2}n$. Write the n th term.

$$\frac{1}{2}n^2 - \frac{3}{2}n + 4$$

Check: Substitute $n = 1$. You get the first term of the sequence.

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Fix it 8 Check with a term

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Substitute $n = 1$ and one more value of n into your n th term. Each must give that term of the sequence. If one does not, look for the slip.

EXAMPLE

Is $3n^2 - 2n + 1$ the n th term of 2, 9, 22, 41, 66, ...

1. $n = 1$: $3 - 2 + 1 = 2 \checkmark$
2. $n = 3$: $3 \times 9 - 6 + 1 = 22 \checkmark$
3. Both match: yes.

YOUR TURN

- 1 Is $2n^2 - 1$ the n th term of 1, 7, 17, 31, 49, ... Substitute $n = 1$ and $n = 3$.

$$\text{Yes: } n = 3 \text{ gives } 17$$

- 2 Sam says the n th term of 4, 9, 18, 31, ... is $n^2 + 3$. Check two terms. Is Sam right?

$$\text{No: } n = 2 \text{ gives } 7, \text{ not } 9$$

- 3 Is $-n^2 + 9n + 1$ the n th term of 9, 15, 19, 21, 21, ... Substitute $n = 1$ and $n = 4$.

$$\text{Yes: } n = 4 \text{ gives } 21$$

Check: Square n before you multiply: $3n^2$ at $n = 4$ is $3 \times 16 = 48$.

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