

Fix it 1 Write it with $\propto$

Proportion · ANSWERS

Read the words. Directly proportional: $y \propto x$. Inversely proportional: 1 over, $y \propto \frac{1}{x}$. The square of x is x^2 , the cube is x^3 , the square root is $\sqrt{x}$.

EXAMPLE

Write with $\propto$:

y is inversely proportional to the square of x .

1. Inversely: 1 over.
2. The square of x : x^2 .
3. $y \propto \frac{1}{x^2}$

YOUR TURN

- 1 Write with $\propto$:
 y is directly proportional to the square of x .

$$y \propto x^2$$

- 2 Write with $\propto$:
The time, T seconds, a pendulum takes to swing is directly proportional to the square root of its length, L cm.

$$T \propto \sqrt{L}$$

- 3 Write with $\propto$:
 c is inversely proportional to the square root of t .

$$c \propto \frac{1}{\sqrt{t}}$$

Check: Read your $\propto$ statement back as words. It says the same as the question.

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Fix it 2 Replace $\propto$ with $= k$

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Swap the $\propto$ for $= k$. k multiplies the x part: $y \propto x^2$ becomes $y = kx^2$. With 1 over, k goes on top: $y \propto \frac{1}{x^2}$ becomes $y = \frac{k}{x^2}$.

EXAMPLE

Write $F \propto \frac{1}{d^2}$ as an equation with k .

1. Swap the $\propto$ for $= k$.
2. d^2 is over 1, so k goes on top.
3. $F = \frac{k}{d^2}$

YOUR TURN

- 1 Write $m \propto r^3$ as an equation with k .

$$m = kr^3$$

- 2 Write $y \propto \frac{1}{x}$ as an equation with k .

$$y = \frac{k}{x}$$

- 3 Write $c \propto \frac{1}{\sqrt{t}}$ as an equation with k .

$$c = \frac{k}{\sqrt{t}}$$

Check: Cover the k with your finger. What is left matches your $\propto$ statement.

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Fix it 3 Substitute

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Substitute the pair of values you are given into the equation with k , each number where its letter is. Then work out the power or root of the number: $5^2 = 25$, $\sqrt{16} = 4$.

EXAMPLE

$y = kx^2$. When $x = 5$, $y = 75$.
Substitute the numbers into the equation.

1. $75 = k \times 5^2$
2. $5^2 = 25$
3. $75 = 25k$

YOUR TURN

- 1 $y = k\sqrt{x}$. When $x = 16$, $y = 20$. Substitute the numbers into the equation.

$$20 = 4k$$

- 2 $F = \frac{k}{d^2}$. When $d = 4$, $F = 3$. Substitute the numbers into the equation.

$$3 = \frac{k}{16}$$

- 3 $c = \frac{k}{\sqrt{t}}$. $c = 2.4$ when $t = 25$. Substitute the numbers into the equation.

$$2.4 = \frac{k}{5}$$

Check: Point to each number. It sits where its letter was, and its power or root is worked out.

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Fix it 4 Find k

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k times a number: divide. $75 = 25k$, so $k = \frac{75}{25} = 3$. k over a number: multiply. $3 = \frac{k}{16}$, so $k = 3 \times 16 = 48$.

EXAMPLE

Find k .

$$4 = \frac{k}{3^2}$$

1. $3^2 = 9$, so $4 = \frac{k}{9}$
2. k is over 9: multiply.
3. $k = 4 \times 9 = 36$

YOUR TURN

- 1 Find k .
 $63 = k \times 3^2$

$$k = 7$$

- 2 Find k .
 $2.5 = \frac{k}{\sqrt{16}}$

$$k = 10$$

- 3 Find k .
 $18 = k \times 2^3$

$$k = 2.25$$

Check: Put k back in. It gives the number on the other side: $36 \div 9 = 4$.

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Fix it 5 Write the formula

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Write the equation again with the number in place of k . Keep the power or root: $k=3$ in $y=kx^2$ gives $y=3x^2$, not $y=3x$.

EXAMPLE

$F = \frac{k}{d^2}$ and $k=48$. Write a formula for F in terms of d .

- Put 48 in place of k .
- Keep the d^2 .
- $F = \frac{48}{d^2}$

YOUR TURN

- 1 $y = kx^3$ and $k=2.5$. Write a formula for y in terms of x .

$$y = 2.5x^3$$

- 2 $T = k\sqrt{L}$ and $k=0.2$. Write a formula for T in terms of L .

$$T = 0.2\sqrt{L}$$

- 3 $c \propto \frac{1}{\sqrt{t}}$. When $t=25$, $c=2.4$, so $k=12$. Write a formula for c in terms of t .

$$c = \frac{12}{\sqrt{t}}$$

Check: Substitute the pair you were given into your formula. It works: $48 \div 4^2 = 3$.

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Fix it 6 Use it

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To find y : substitute the new x and work out its power or root first (BIDMAS). To find x : substitute y , get the x part on its own, then undo it: square root a square, square a square root.

EXAMPLE

$y = 3x^2$. Find y when $x=4$, then x when $y=108$.

- $y = 3 \times 4^2 = 3 \times 16 = 48$
- $108 = 3x^2$, so $x^2 = 36$
- $x = \sqrt{36} = 6$

YOUR TURN

- 1 $F = \frac{48}{d^2}$. Find F when $d=8$.

$$F = 0.75$$

- 2 $T = 0.2\sqrt{L}$. Find L when $T=1.2$.

$$L = 36$$

- 3 $F = \frac{48}{d^2}$. Find d when $F=0.12$.

$$d = 20$$

Check: Substitute your answer back into the formula. You get the number you started with.

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Direct: one goes up, the other goes up. **Inverse:** one goes up, the other goes down. A power makes the change bigger: in $y \propto x^2$, $x \times 2$ gives $y \times 4$, because $2^2 = 4$.

EXAMPLE

$y \propto x^2$. When $x = 3$, $y = 18$. Jo gets $y = 72$ when $x = 6$. Does it make sense?

1. Direct: x went up, so y goes up.
2. $x \times 2$, so $y \times 2^2$, which is $y \times 4$.
3. $18 \times 4 = 72$. Yes.

YOUR TURN

- 1 $y \propto x$. When $x = 5$, $y = 12$. Ana says when $x = 10$, $y = 6$. Does that make sense?

No: x went up, so y goes up

- 2 $y \propto \frac{1}{x^2}$. When $x = 2$, $y = 36$. Ben says when $x = 6$, $y = 4$. Does that make sense?

Yes: $x \times 3$, so $y \times \frac{1}{9}$

- 3 $y \propto x^2$. When $x = 5$, $y = 20$. Cal says when $x = 10$, $y = 40$. Does that make sense?

No: $x \times 2$, so $y \times 4 = 80$

Check: Work it out with the formula as well. Both ways give the same answer.

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