

Tree diagrams and frequency trees

Foundation and Higher tiers | Total marks: 92

Marks

- M** Method mark, for a correct method. It can be implied by a correct answer.
- A** Accuracy mark, for a correct answer. It needs the method mark before it.
- B** A mark for a correct value or statement, with or without working.
- ft** Follow through: the mark is given for using an earlier wrong answer correctly.
- oe** Or equivalent.
- [a, b]** Any value from a to b inclusive.

Questions

Q	Skill	Spec	Tier	Grade	Calculator	Marks
1	Completing a frequency tree	P1	Foundation	3	No	4
2	Frequency trees with fractions	P1	Foundation	4	No	5
3	Completing a tree diagram	P8	Common	5	No	4
4	The multiplication rule	P8 P4	Common	5	No	4
5	Tree diagrams: exactly one	P8	Common	6	No	3
6	Independent events in reverse	P8	Common	6	No	3
7	Conditional probability from a frequency tree	P9 P1	Higher	6	No	2
8	Dependent events on a tree diagram	P8	Common	6	No	4
9	Without replacement	P8	Common	7	No	4
10	Three events without replacement	P8	Common	8	No	4
11	Taking turns until the first success	P8	Common	8	No	4
12	Forming an equation from a probability	P8 A21 A18	Higher	9	No	6
13	Reading a frequency tree	P1 P3	Foundation	2	Yes	3
14	Frequency trees with percentages	P1	Foundation	3	Yes	5
15	Checking and using a tree diagram	P8	Common	4	Yes	3
16	Expected results of combined events	P8 P2	Common	5	Yes	3

Q	Skill	Spec	Tier	Grade	Calculator	Marks
17	At least one, from three events	P8	Common	6	Yes	3
18	At least one of two independent events	P8 P2	Common	6	Yes	4
19	At least one, without replacement	P8	Common	7	Yes	4
20	Conditional probability from a tree diagram	P9 P8	Higher	8	Yes	4
21	Working backwards, then a conditional probability	P9 P8	Higher	8	Yes	5
22	Dependent events over three stages, then a conditional	P9 P8	Higher	9	Yes	5
23	Forming a quadratic from a tree diagram	P8 A18	Higher	9	Yes	6
	Total					92

Question 1 P1 · Foundation · Grade 3 · 4 marks

Q	Answer	Mark	Comments
1(a)	adult 36, child 54 adult: hired 20, did not hire 16 child: hired 41, did not hire 13	B3	B2 for 4 or 5 correct, B1 for 2 or 3 correct
1(b)	$\frac{41}{90}$	B1ft	oe, ft their tree

Question 2 P1 · Foundation · Grade 4 · 5 marks

Q	Answer	Mark	Comments
2(a)	breakfast 60 and no breakfast 100	B1	
	tea 50 and no tea 10	B1ft	ft their 60
	tea 40 and no tea 60	B1ft	ft their 100
2(b)	$\frac{50 + 40}{160}$	M1	oe, ft their tree
	$\frac{9}{16}$	A1	

Question 3 P8 · Common · Grade 5 · 4 marks

Q	Answer	Mark	Comments
3(a)	0.65 for no rain	B1	
	0.8, 0.2 and 0.8 in the correct places	B1	
3(b)	0.35×0.2	M1	oe
	0.07	A1	oe, e.g. $\frac{7}{100}$

Question 4 P8 P4 · Common · Grade 5 · 4 marks

Q	Answer	Mark	Comments
4(a)	a probability cannot be more than 1	B1	oe
4(b)	0.42	B1	oe
4(c)	0.3×0.4	M1	oe
	0.12	A1	oe

Question 5 P8 · Common · Grade 6 · 3 marks

Q	Answer	Mark	Comments
5	$\frac{3}{5} \times \frac{2}{5}$ or $\frac{6}{25}$	M1	oe, one correct product
	$\frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{3}{5}$	M1	oe
	$\frac{12}{25}$	A1	oe, e.g. 0.48

Question 6 P8 · Common · Grade 6 · 3 marks

Q	Answer	Mark	Comments
6	$0.3 \times p = 0.12$	M1	oe
	$0.12 \div 0.3$ or 0.4	M1	oe
	0.6	A1	

Question 7 P9 P1 · Higher · Grade 6 · 2 marks

Q	Answer	Mark	Comments
7	$54 + 46$ or 100	M1	the number who passed, or $\frac{54}{\text{their } 100}$
	$\frac{27}{50}$	A1	oe, e.g. $\frac{54}{100}$ or 0.54

Question 8 P8 · Common · Grade 6 · 4 marks

Q	Answer	Mark	Comments
8(a)	0.6 for away	B1	
	0.35, then 0.3 and 0.7 in the correct places	B1	
8(b)	$0.4 \times 0.65 + 0.6 \times 0.3$	M1	oe, ft their tree
	0.44	A1	oe

Question 9 P8 · Common · Grade 7 · 4 marks

Q	Answer	Mark	Comments
9	$\frac{8}{14}$ or $\frac{5}{14}$ seen as a second probability	B1	oe, shows the second pick is without replacement
	$\frac{9}{15} \times \frac{8}{14}$ or $\frac{6}{15} \times \frac{5}{14}$	M1	oe, one correct product
	$\frac{9}{15} \times \frac{8}{14} + \frac{6}{15} \times \frac{5}{14}$	M1	oe
	$\frac{17}{35}$	A1	oe, e.g. $\frac{102}{210}$

Question 10 P8 · Common · Grade 8 · 4 marks

Q	Answer	Mark	Comments
10	a product of three fractions with denominators 10, 9 and 8	M1	shows the picks are without replacement
	$\frac{4}{10} \times \frac{6}{9} \times \frac{5}{8}$ or $\frac{120}{720}$	M1	oe, one order
	$3 \times \frac{120}{720}$	M1	oe, or the three orders added
	$\frac{1}{2}$	A1	oe, e.g. $\frac{360}{720}$

Question 11 P8 · Common · Grade 8 · 4 marks

Q	Answer	Mark	Comments
11	$\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}$	M1	oe, Priya wins with her second envelope, $\frac{60}{336}$ or $\frac{5}{28}$
	$\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{2}{5} \times \frac{3}{4}$	M1	oe, Priya wins with her third envelope, $\frac{3}{56}$
	$\frac{3}{8} + \text{their } \frac{5}{28} + \text{their } \frac{3}{56}$	M1	oe, the three ways for Priya to win added, with no others
	$\frac{17}{28}$	A1	oe, e.g. $\frac{34}{56}$

Question 12 P8 A21 A18 · Higher · Grade 9 · 6 marks

Q	Answer	Mark	Comments
12(a)	$\frac{5}{n+5} \times \frac{n}{n+4}$	M1	oe, one order
	$2 \times \frac{5n}{(n+5)(n+4)} = \frac{10}{21}$	M1	oe, both orders in an equation
	$210n = 10(n^2 + 9n + 20)$	M1	oe, fractions cleared and brackets expanded
	$21n = n^2 + 9n + 20$, so $n^2 - 12n + 20 = 0$	A1	with no errors seen
12(b)	$(n-2)(n-10) = 0$	M1	oe, or $n=2$ and $n=10$
	10 only	A1	2 rejected as it gives fewer blue pens than red pens

Question 13 P1 P3 · Foundation · Grade 2 · 3 marks

Q	Answer	Mark	Comments
13(a)	999	B1	
13(b)	$\frac{255}{1865}$	M1	oe
	0.14	A1	accept 0.137 or [0.1367, 0.1368]

Question 14 P1 · Foundation · Grade 3 · 5 marks

Q	Answer	Mark	Comments
14(a)	diesel 540 and petrol 960	B1	
	diesel: failed 81 and passed 459	B1ft	ft their 540
	petrol: failed 112 and passed 848	B1ft	ft their 960
14(b)	$\frac{81 + 112}{1500}$	M1	oe, ft their tree
	0.129	A1ft	accept $\frac{193}{1500}$ or 0.1286...

Question 15 P8 · Common · Grade 4 · 3 marks

Q	Answer	Mark	Comments
15(a)	0.52 (after right) should be 0.42	B1	oe, e.g. 0.58 and 0.52 add to more than 1
15(b)	0.73×0.42	M1	oe
	0.3066	A1	oe

Question 16 P8 P2 · Common · Grade 5 · 3 marks

Q	Answer	Mark	Comments
16	$0.4 \times \frac{1}{6}$	M1	oe, e.g. $\frac{1}{15}$ or 0.066...
	their $\frac{1}{15} \times 300$	M1	oe
	20	A1	

Question 17 P8 · Common · Grade 6 · 3 marks

Q	Answer	Mark	Comments
17	0.55	M1	the probability of not green
	$1 - 0.55^3$	M1	oe, or adds all 7 correct products
	0.833625	A1	accept 0.834 or 0.83

Question 18 P8 P2 · Common · Grade 6 · 4 marks

Q	Answer	Mark	Comments
18	0.965×0.948 or 0.91482	M1	oe, or one correct product for a stopping route, e.g. 0.035×0.948
	$1 - 0.91482$ or 0.08518	M1	oe, or all three stopping routes added
	their 0.08518×250	M1	oe
	21	A1	accept 21.295 or 21.3

Question 19 P8 · Common · Grade 7 · 4 marks

Q	Answer	Mark	Comments
19	$\frac{25}{29}$ or $\frac{24}{28}$ seen	B1	oe, a later pick without replacement
	$\frac{26}{30} \times \frac{25}{29} \times \frac{24}{28}$ or 0.6404...	M1	oe, none cracked
	1 – their 0.6404	M1	oe, or all the routes with at least one cracked added
	0.360	A1	accept 0.360 or [0.3596, 0.3597]

Question 20 P9 P8 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
20	0.6×0.03 or 0.018 or 0.4×0.05 or 0.02	M1	oe
	$0.018 + 0.02$ or 0.038	M1	oe, the probability of a mistake
	$\frac{0.02}{0.038}$	M1	oe, ft their values
	0.526	A1	accept $\frac{10}{19}$ or 0.5263...

Question 21 P9 P8 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
21(a)	$(1 - p)^2 = 1 - 0.8704$ or 0.1296	M1	oe
	$1 - p = \sqrt{0.1296}$ or 0.36	M1	oe
	0.64	A1	
21(b)	$\frac{0.64^2}{0.8704}$ or $\frac{0.4096}{0.8704}$	M1	oe, ft their p
	$\frac{8}{17}$	A1	oe, accept 0.4706...

Question 22 P9 P8 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
22(a)	one correct product for two wins and one loss	M1	eg $0.6 \times 0.7 \times 0.3 = 0.126$ or $0.6 \times 0.3 \times 0.45 = 0.081$ or $0.4 \times 0.45 \times 0.7 = 0.126$
	$0.126 + 0.081 + 0.126$	M1	oe, all three products added and no others
	0.333	A1	oe
22(b)	$\frac{0.126 + 0.081}{\text{their } 0.333}$	M1	oe, ft their (a)
	0.622	A1	accept $\frac{23}{37}$ or 0.6216...

Question 23 P8 A18 · Higher · Grade 9 · 6 marks

Q	Answer	Mark	Comments
23(a)	$p(p + 0.15) = 0.3$	M1	oe
	$\frac{-0.15 \pm \sqrt{(0.15^2 + 4 \times 0.3)}}{2}$	M1	oe, correct substitution for $p^2 + 0.15p - 0.3 = 0$
	0.478	A1	accept [0.4778, 0.4779], the negative root rejected
23(b)	$p(1 - (p + 0.15))$ or $(1 - p)(p - 0.2)$ with their p	M1	oe, e.g. 0.478×0.372
	$0.478 \times 0.372 + 0.522 \times 0.278$	M1	oe, both products added
	0.323	A1	accept [0.322, 0.324]