

Simultaneous equations

Foundation and Higher tiers | Total marks: 87

Marks

- M** Method mark, for a correct method. It can be implied by a correct answer.
- A** Accuracy mark, for a correct answer. It needs the method mark before it.
- B** A mark for a correct value or statement, with or without working.
- ft** Follow through: the mark is given for using an earlier wrong answer correctly.
- oe** Or equivalent.
- [a, b]** Any value from a to b inclusive.

Questions

Q	Skill	Spec	Tier	Grade	Calculator	Marks
1	Eliminating by subtracting	A19	Common	4	No	3
2	Checking a solution by substitution	A19	Common	4	No	1
3	Multiplying one equation	A19	Common	5	No	4
4	Forming equations in context	A21 A19	Common	5	No	5
5	Solving with a graph	A19	Common	5	No	2
6	Spotting a mistake	A19	Common	5	No	3
7	Multiplying both equations	A19	Common	6	No	4
8	Equations from a rectangle	A21 A19	Common	6	No	4
9	Linear and quadratic	A19 A18	Higher	7	No	5
10	Perimeter and area of a rectangle	A21 A19	Higher	7	No	5
11	A circle and a line	A19 A18	Higher	8	No	5
12	Showing a line misses a circle	A19 A18	Higher	8	No	4
13	Intersecting tangents, then a square	A16 A9 A19	Higher	9	No	6
14	Fixed charge and hourly rate	A21 A19	Common	6	Yes	4
15	Forming equations: a total and a takings	A21 A19	Common	6	Yes	4
16	Solving with decimal coefficients	A19 A9	Common	6	Yes	5

Q	Skill	Spec	Tier	Grade	Calculator	Marks
17	Forming and solving from two bills	A21 A19	Common	7	Yes	5
18	Forming equations from percentages	A21 A19 R9	Common	7	Yes	5
19	Forming equations from speeds and times	A21 A19 R11	Common	7	Yes	4
20	Simultaneous equations from points on a curve	A19 A12 A21	Common	7	Yes	4
21	A reciprocal curve and a line, by the formula	A19 A18	Higher	8	Yes	5
	Total					87

Question 1 A19 · Common · Grade 4 · 3 marks

Q	Answer	Mark	Comments
1	$2x = 10$	M1	subtracts the equations, or another correct method to eliminate one letter
	$x = 5$	A1	or $y = 4$
	$y = 4$	A1ft	ft their first value substituted correctly

Question 2 A19 · Common · Grade 4 · 1 mark

Q	Answer	Mark	Comments
2	$x = 5, y = -3$	B1	

Question 3 A19 · Common · Grade 5 · 4 marks

Q	Answer	Mark	Comments
3	$6x - 3y = 15$ or $4x - 2y = 10$	M1	one equation multiplied so a letter can be eliminated, or $y = 2x - 5$ substituted
	$10x = 40$ or $5y = 15$	M1	oe, a correct elimination
	$x = 4$	A1	or $y = 3$
	$y = 3$	A1ft	ft their first value

Question 4 A21 A19 · Common · Grade 5 · 5 marks

Q	Answer	Mark	Comments
4	$3m + 2p = 61$ and $2m + 5p = 70$	M1	oe, any letters
	$6m + 4p = 122$ and $6m + 15p = 210$	M1	oe, a correct method to eliminate one letter
	mug £15	A1	or plate £8
	plate £8	A1	or mug £15
	£39	A1ft	ft their prices

Question 5 A19 · Common · Grade 5 · 2 marks

Q	Answer	Mark	Comments
5	$x = 1.5$	B1	accept [1.4, 1.6]
	$y = 2.5$	B1	accept [2.4, 2.6]

Question 6 A19 · Common · Grade 5 · 3 marks

Q	Answer	Mark	Comments
6(a)	$3y - (-y) = 4y$, not $2y$	B1	oe, such as 'he took y away instead of adding it'
6(b)	$4y = 12$ or $y = 3$	M1	
	$x = 2, y = 3$	A1	

Question 7 A19 · Common · Grade 6 · 4 marks

Q	Answer	Mark	Comments
7	$25a + 10b = 5$ and $6a - 10b = 88$	M1	oe, both equations multiplied to match the coefficients of one letter
	$31a = 93$ or $31b = -217$	M1	oe, a correct elimination
	$a = 3$	A1	or $b = -7$
	$b = -7$	A1ft	ft their first value

Question 8 A21 A19 · Common · Grade 6 · 4 marks

Q	Answer	Mark	Comments
8	$3x + 2y = 17$ and $x - y = 4$	M1	opposite sides put equal
	$5x = 25$ or $5y = 5$	M1	oe, a correct method to eliminate one letter
	$x = 5$	A1	
	$y = 1$	A1	

Question 9 A19 A18 · Higher · Grade 7 · 5 marks

Q	Answer	Mark	Comments
9	$x^2 - 3x + 2 = 2x - 4$	M1	the two expressions for y put equal
	$x^2 - 5x + 6 = 0$	M1	oe
	$(x - 2)(x - 3)$	M1	or a correct use of the formula
	$x = 2$ and $x = 3$	A1	
	$x = 2, y = 0$ and $x = 3, y = 2$	A1	correctly paired

Question 10 A21 A19 · Higher · Grade 7 · 5 marks

Q	Answer	Mark	Comments
10	$2x + 2y = 23$ and $xy = 30$	M1	oe, any letters
	$x(11.5 - x) = 30$	M1	oe, one letter eliminated
	$2x^2 - 23x + 60 = 0$	M1	oe, a three-term quadratic
	$(2x - 15)(x - 4)$ or 7.5 and 4	A1	or a correct use of the formula
	7.5	A1	

Question 11 A19 A18 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
11	$x^2 + (x + 2)^2 = 34$	M1	a correct substitution
	$2x^2 + 4x - 30 = 0$ or $x^2 + 2x - 15 = 0$	M1	oe, expanded and simplified
	$(x + 5)(x - 3)$	M1	or a correct use of the formula
	$x = -5$ and $x = 3$	A1	
	$x = -5, y = -3$ and $x = 3, y = 5$	A1	correctly paired

Question 12 A19 A18 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
12	$x^2 + (2x + 7)^2 = 9$	M1	a correct substitution
	$5x^2 + 28x + 40 = 0$	M1	oe
	$28^2 - 4 \times 5 \times 40$	M1	oe, or an attempt at the formula or completing the square
	$\sqrt{(-16)},$ or $b^2 - 4ac = -16 < 0$, so there are no real solutions and the line and circle do not meet	A1	a full method and a conclusion. Alternative: the distance from (0, 0) to the line is $\frac{7}{\sqrt{5}} = 3.13... > 3$, the radius, so they do not meet (M3 A1)

Question 13 A16 A9 A19 · Higher · Grade 9 · 6 marks

Q	Answer	Mark	Comments
13(a)	gradient of OP = 7 so the gradient of the tangent at P is $-\frac{1}{7}$	M1	oe
	$y - 7 = -\frac{1}{7}(x - 1)$ or $x + 7y = 50$	M1	oe, the tangent at P, ft their gradient
	$y + 1 = 7(x - 7)$ or $y = 7x - 50$, and an attempt to solve with their tangent at P	M1	oe, the tangent at Q has gradient 7
	(8, 6)	A1	
13(b)	OP and OQ are perpendicular: gradients 7 and $-\frac{1}{7}$ multiply to -1	B1	oe, or angle POQ = 90° from $OP^2 + OQ^2 = PQ^2$ (50 + 50 = 100)
	a complete reason: OP = OQ = $\sqrt{50}$ (radii) and the angles at P and Q are 90° (a tangent is perpendicular to the radius), so OPTQ has four right angles and two equal adjacent sides	B1	oe, or all four sides $\sqrt{50}$ ($PT^2 = 7^2 + 1^2$) with one right angle

Question 14 A21 A19 · Common · Grade 6 · 4 marks

Q	Answer	Mark	Comments
14	$f + 3h = 27.60$ and $f + 5h = 39.20$	M1	oe, any letters
	$2h = 11.60$ or $h = 5.80$	M1	oe
	fixed charge £10.20 and £5.80 an hour	A1	
	£56.60	A1	

Question 15 A21 A19 · Common · Grade 6 · 4 marks

Q	Answer	Mark	Comments
15	$s + c = 420$ and $24.50s + 18.75c = 9341.25$	M1	oe, any letters
	$24.50s + 18.75(420 - s) = 9341.25$	M1	oe, a correct method to eliminate one letter
	$5.75s = 1466.25$	M1	oe
	255	A1	

Question 16 A19 A9 · Common · Grade 6 · 5 marks

Q	Answer	Mark	Comments
16(a)	$4.8x - 1.5y = 2.55$	M1	oe, the second equation multiplied by 3, or $y = 3.2x - 1.7$
	$7.2x = 11.7$	M1	oe, a correct elimination or substitution
	$x = 1.625$	A1	
	$y = 3.5$	A1ft	ft their x
16(b)	2.15	B1ft	ft their P, accept 2.15 or [2.153, 2.154]

Question 17 A21 A19 · Common · Grade 7 · 5 marks

Q	Answer	Mark	Comments
17(a)	$31s + 284u = 8570$ and $30s + 252u = 7734$	M1	oe, in pence or in pounds
	$930s + 8520u = 257100$ and $930s + 7812u = 239754$	M1	oe, a correct method to eliminate one letter, or $708u = 17346$
	$u = 24.5$	A1	or $s = 52$
	$s = 52$	A1	or $u = 24.5$, both correct
17(b)	£72.47	B1ft	ft their s and u

Question 18 A21 A19 R9 · Common · Grade 7 · 5 marks

Q	Answer	Mark	Comments
18	$p + q = 600$	M1	oe, any letters
	$0.925p + 0.8q = 525$ or $0.925p + 0.8q = 0.875 \times 600$	M1	oe
	$0.125p = 45$ or $0.125q = 30$	M1	oe, a correct method to eliminate one letter
	P 360 g	A1	or Q 240 g
	Q 240 g	A1ft	ft 600 – their P

Question 19 A21 A19 R11 · Common · Grade 7 · 4 marks

Q	Answer	Mark	Comments
19	$0.75c + \frac{1}{3}w = 13.4$ or $0.5c + \frac{5}{6}w = 11.5$	M1	oe, times in hours, any letters
	both equations correct, with times in hours	M1	oe, such as $45c + 20w = 804$ and $30c + 50w = 690$
	a correct method to eliminate one letter	M1	such as $90c + 40w = 1608$ and $90c + 150w = 2070$, so $110w = 462$
	4.2	A1	

Question 20 A19 A12 A21 · Common · Grade 7 · 4 marks

Q	Answer	Mark	Comments
20	$4 + 2p + q = 3.6$ or $2p + q = -0.4$	M1	oe
	$2.25 - 1.5p + q = 7.1$ or $-1.5p + q = 4.85$	M1	oe
	$3.5p = -5.25$	M1	oe, a correct method to eliminate one letter from their equations
	$p = -1.5$ and $q = 2.6$	A1	

Question 21 A19 A18 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
21	$x(x + 2) = 6$	M1	a correct substitution
	$x^2 + 2x - 6 = 0$	M1	oe
	$\frac{-2 \pm \sqrt{(2^2 - 4 \times 1 \times -6)}}{2 \times 1}$	M1	oe, or $(x+1)^2 = 7$
	$x = 1.65$ and $x = -3.65$	A1	accept [1.645, 1.646] and [-3.646, -3.645]
	$x = 1.65, y = 3.65$ and $x = -3.65, y = -1.65$	A1	y values accept [3.645, 3.646] and [-1.646, -1.645], correctly paired