

Quadratic equations

Foundation and Higher tiers | Total marks: 110

Marks

- M** Method mark, for a correct method. It can be implied by a correct answer.
- A** Accuracy mark, for a correct answer. It needs the method mark before it.
- B** A mark for a correct value or statement, with or without working.
- ft** Follow through: the mark is given for using an earlier wrong answer correctly.
- oe** Or equivalent.
- [a, b]** Any value from a to b inclusive.

Questions

Q	Skill	Spec	Tier	Grade	Calculator	Marks
1	Expanding two brackets	A4	Common	3	No	2
2	Factorising and solving a quadratic	A4 A18	Common	4	No	3
3	Difference of two squares	A4	Common	5	No	3
4	Spot the error when solving a quadratic	A18 A4	Common	5	No	4
5	Reading solutions from a quadratic graph	A11 A18	Common	5	No	4
6	Forming and solving a quadratic	A18 A4	Common	5	No	5
7	A quadratic from its solutions	A18 A4	Common	5	No	3
8	Rearranging and solving a quadratic with $a \neq 1$	A18 A4	Higher	6	No	3
9	Pythagoras leading to a quadratic	A18 G20	Common	6	No	4
10	A quadratic from its turning point	A18 A11	Higher	7	No	3
11	Solving a quadratic in surd form	A18 N8	Higher	8	No	4
12	Completing the square to show there are no solutions	A18 A11	Higher	8	No	3
13	Completing the square with $a \neq 1$	A18 A11	Higher	8	No	4
14	Proving a line is a tangent to a circle	A19 A16 A18	Higher	9	No	5
15	The quadratic formula	A18	Higher	6	Yes	3
16	Forming a quadratic and using the formula	A18 A4	Higher	7	Yes	6

Q	Skill	Spec	Tier	Grade	Calculator	Marks
17	Forming a quadratic from a surface area	A18 G17 A21	Higher	8	Yes	5
18	Pythagoras in 3D leading to a quadratic	A18 G20	Higher	8	Yes	6
19	Where two quadratic curves meet	A18 A19	Higher	8	Yes	5
20	When one sequence overtakes another	A18 A25	Higher	8	Yes	4
21	The length of a chord cut off by a line	A19 A18 G20	Higher	9	Yes	5
22	Takings: a greatest value and a range of prices	A18 A11 A21	Higher	9	Yes	6
23	The cosine rule leading to a quadratic	A18 G22	Higher	9	Yes	6
24	Similar rectangles leading to a quadratic	A18 G19	Higher	9	Yes	4
25	Compound interest leading to a quadratic	A18 R16 A21	Higher	9	Yes	5
26	Percentage change leading to a quadratic	A18 R9 A21	Higher	9	Yes	5
	Total					110

Question 1 A4 · Common · Grade 3 · 2 marks

Q	Answer	Mark	Comments
1	three of x^2 , $7x$, $-3x$, -21	M1	four terms with three correct, or a correct expansion with one error
	$x^2 + 4x - 21$	A1	

Question 2 A4 A18 · Common · Grade 4 · 3 marks

Q	Answer	Mark	Comments
2(a)	$(x \pm 3)(x \pm 8)$	M1	or $(x + a)(x + b)$ with $ab = 24$ or $a + b = -11$
	$(x - 3)(x - 8)$	A1	either order
2(b)	3 and 8	B1ft	ft their factors

Question 3 A4 · Common · Grade 5 · 3 marks

Q	Answer	Mark	Comments
3(a)	$(x + 9)(x - 9)$	B1	either order
3(b)	$x = 100$ or $100^2 - 9^2$ or $10\,000 - 81$	M1	
	9919	A1	no marks for a long multiplication with no use of part (a)

Question 4 A18 A4 · Common · Grade 5 · 4 marks

Q	Answer	Mark	Comments
4(a)	the equation must equal 0 before factorising	B1	oe, e.g. two numbers that multiply to 28 do not have to be 28 and 1
4(b)	$x^2 + 3x - 28 = 0$	M1	
	$(x + 7)(x - 4)$	M1	oe, or a correct use of the formula
	-7 and 4	A1	

Question 5 A11 A18 · Common · Grade 5 · 4 marks

Q	Answer	Mark	Comments
5(a)	(0, -2)	B1	
5(b)	one of $[-0.7, -0.4]$ or $[3.4, 3.7]$	B1	
	both $[-0.7, -0.4]$ and $[3.4, 3.7]$	B1	
5(c)	the lowest point of the graph is at about -4.25, so the graph never reaches $y = -5$	B1	oe, accept a minimum value in $[-4.5, -4]$; or the line $y = -5$ drawn and does not meet the curve

Question 6 A18 A4 · Common · Grade 5 · 5 marks

Q	Answer	Mark	Comments
6(a)	$x(x + 3) = 10$	M1	oe
	$x^2 + 3x = 10$ so $x^2 + 3x - 10 = 0$	A1	a correct expansion leading to the given equation
6(b)	$(x + 5)(x - 2)$	M1	oe
	$x = 2$	A1	may also give -5
	5	A1	must reject -5 or use only 2

Question 7 A18 A4 · Common · Grade 5 · 3 marks

Q	Answer	Mark	Comments
7	$(x + 3)(x - 7)$	M1	oe, or $9 - 3b + c = 0$ and $49 + 7b + c = 0$
	$b = -4$	A1	
	$c = -21$	A1	

Question 8 A18 A4 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
8	$3x^2 + 10x - 8 = 0$	M1	oe, all terms on one side
	$(3x - 2)(x + 4)$	M1	oe, or a correct substitution into the formula
	$\frac{2}{3}$ and -4	A1	accept 0.67 or better for $\frac{2}{3}$

Question 9 A18 G20 · Common · Grade 6 · 4 marks

Q	Answer	Mark	Comments
9	$x^2 + (x + 7)^2 = (x + 8)^2$	M1	oe
	$x^2 - 2x - 15 = 0$	M1	oe, collects terms to a quadratic equal to 0
	$(x - 5)(x + 3)$	M1	oe, or $x = 5$
	30	A1	

Question 10 A18 A11 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
10	$y = (x - 4)^2 - 11$	M1	oe
	$x^2 - 8x + 16 - 11$	M1	a correct expansion
	$p = -8$ and $q = 5$	A1	

Question 11 A18 N8 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
11	$(x-2)^2 - 4 - 14 = 0$	M1	oe, or $\frac{4 \pm \sqrt{(16+56)}}{2}$
	$(x-2)^2 = 18$	A1	oe, or $\frac{4 \pm \sqrt{72}}{2}$
	$\sqrt{18} = 3\sqrt{2}$	M1	or $\sqrt{72} = 6\sqrt{2}$
	$2 \pm 3\sqrt{2}$	A1	

Question 12 A18 A11 · Higher · Grade 8 · 3 marks

Q	Answer	Mark	Comments
12	$(x-3)^2$	M1	
	$(x-3)^2 + 2$	A1	
	$(x-3)^2 \geq 0$ so $(x-3)^2 + 2 \geq 2$, which is never 0	A1	oe, a full explanation

Question 13 A18 A11 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
13(a)	$2(x^2 + 6x) + 5$	M1	oe
	$2(x+3)^2$	M1	or $a=2$ and $b=3$
	$2(x+3)^2 - 13$	A1	
13(b)	$x = -3$	B1ft	ft their b

Question 14 A19 A16 A18 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
14(a)	$(10-3y)^2 + y^2 = 10$	M1	oe, a correct substitution, e.g. $9x^2 + (10-x)^2 = 90$
	$10y^2 - 60y + 90 = 0$	M1	oe, e.g. $y^2 - 6y + 9 = 0$ or $x^2 - 2x + 1 = 0$
	$(y-3)^2 = 0$	M1	oe, or $b^2 - 4ac = 0$
	only one solution, so the line meets the circle at exactly one point, so it is a tangent	A1	a full method and a conclusion
14(b)	(1, 3)	B1ft	ft their repeated root

Question 15 A18 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
15	$\frac{5 \pm \sqrt{((-5)^2 - 4 \times 3 \times -4)}}{2 \times 3}$	M1	oe, allow one sign error
	$\frac{5 \pm \sqrt{73}}{6}$	A1	oe
	-0.59 and 2.26	A1	accept -0.59 or [-0.591, -0.590]; accept 2.26 or [2.257, 2.258]

Question 16 A18 A4 · Higher · Grade 7 · 6 marks

Q	Answer	Mark	Comments
16(a)	$(8 + 2x)(5 + 2x)$ or $(8 + 2x)(5 + 2x) = 60$	M1	oe, the length and width of the card
	$40 + 16x + 10x + 4x^2$	M1	a correct expansion
	$4x^2 + 26x - 20 = 0$ and division by 2	A1	a full method to the given equation
16(b)	$\frac{-13 \pm \sqrt{(13^2 - 4 \times 2 \times -10)}}{2 \times 2}$	M1	oe
	$\frac{-13 \pm \sqrt{249}}{4}$	A1	oe
	0.695	A1	must reject the negative solution. accept 0.695 or [0.6949, 0.6950]

Question 17 A18 G17 A21 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
17	$2\pi r^2$ or $2\pi r(r + 3)$	M1	one correct area: the two circles or the curved surface
	$2\pi r^2 + 2\pi r(r + 3) = 500$	M1	oe
	$4\pi r^2 + 6\pi r - 500 = 0$	M1	oe, e.g. $12.57r^2 + 18.85r - 500 = 0$
	$\frac{-6\pi \pm \sqrt{((6\pi)^2 - 4 \times 4\pi \times -500)}}{2 \times 4\pi}$	M1	oe, a correct substitution of their values into the formula
	5.60	A1	accept 5.60 or [5.602, 5.603]. Rejects the negative solution

Question 18 A18 G20 · Higher · Grade 8 · 6 marks

Q	Answer	Mark	Comments
18	$(2x)^2 + x^2 + (x+1)^2$	M1	oe, with the width x , any letter
	$6x^2 + 2x - 323 = 0$	M1	oe, from setting their expression equal to 18^2
	$\frac{-2 \pm \sqrt{(2^2 - 4 \times 6 \times -323)}}{2 \times 6}$	M1	oe
	$x = 7.17$	A1	accept [7.172, 7.173]
	$2x \times x \times (x+1)$ for their positive x	M1	oe
	841	A1	accept [840, 841]

Question 19 A18 A19 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
19	$2x^2 - 3x = x^2 + x + 3$	M1	
	$x^2 - 4x - 3 = 0$	M1	oe
	$\frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times -3}}{2 \times 1}$	M1	oe, or $(x-2)^2 = 7$
	$x = -0.65$ and 4.65	A1	accept [-0.646, -0.645] and [4.645, 4.646]
	P(-0.65, 2.77), Q(4.65, 29.23)	A1	y values accept [2.77, 2.80] and [29.22, 29.30], correctly paired

Question 20 A18 A25 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
20	$2n^2 - 5n + 3 = 60n + 100$	M1	oe, or an inequality
	$2n^2 - 65n - 97 = 0$	M1	oe
	$n = 33.93$	M1	a correct use of the formula, accept [33.92, 33.94]; or the 33rd terms 2016 and 2080 and the 34th terms 2145 and 2140
	34	A1	

Question 21 A19 A18 G20 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
21	$x^2 + (2x - 3)^2 = 30$	M1	a correct substitution
	$5x^2 - 12x - 21 = 0$	M1	oe
	$x = 3.575$ and -1.175	A1	from a correct use of the formula; accept 2 decimal places or better
	$\sqrt{((3.57 - -1.17)^2 + (4.15 - -5.35)^2)}$	M1	oe, Pythagoras with their two points
	10.6	A1	accept [10.59, 10.63]

Question 22 A18 A11 A21 · Higher · Grade 9 · 6 marks

Q	Answer	Mark	Comments
22(a)	$-20(p - 3.75)^2 + 281.25$ or $p = 3.75$	M1	oe, e.g. halfway between the roots 0 and 7.5
	£281.25	A1	
22(b)	$p(150 - 20p) = 275$	M1	oe, e.g. $20p^2 - 150p + 275 = 0$ or $4p^2 - 30p + 55 = 0$
	$\frac{150 \pm \sqrt{(150^2 - 4 \times 20 \times 275)}}{2 \times 20}$	M1	oe
	3.191 and 4.309	A1	accept 3.19 and 4.31 for this mark
	lowest £3.20 and highest £4.30	A1	both, rounded inwards; £3.19 and £4.31 each take £274.98, less than £275

Question 23 A18 G22 · Higher · Grade 9 · 6 marks

Q	Answer	Mark	Comments
23(a)	$15^2 = x^2 + (x + 4)^2 - 2 \times x \times (x + 4) \times \cos 70^\circ$	M1	oe
	$1.316x^2 + 5.264x - 209 = 0$	M1	oe, a three-term quadratic, accept coefficients to 2 decimal places
	a correct substitution of their values into the formula	M1	
	10.8	A1	accept 10.8 or [10.76, 10.77]
23(b)	$\frac{1}{2} \times x \times (x + 4) \times \sin 70^\circ$ for their x	M1	
	74.6	A1	accept [74.6, 75.2]

Question 24 A18 G19 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
24	$\frac{L}{12} = \frac{12}{L-12}$	M1	oe, with the length L ; matching the longer sides and the shorter sides
	$L^2 - 12L - 144 = 0$	M1	oe
	$\frac{12 \pm \sqrt{(12^2 - 4 \times 1 \times -144)}}{2}$	M1	oe, or $(L-6)^2 = 180$
	19.4	A1	accept 19.4 or [19.41, 19.42]

Question 25 A18 R16 A21 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
25(a)	$(2000m + 500) \times m$	M1	oe, the amount at the end of the second year
	$2000m^2 + 500m = 2700$ leading to the given equation	A1	a full method with no errors
25(b)	$\frac{-5 \pm \sqrt{(5^2 - 4 \times 20 \times -27)}}{2 \times 20}$	M1	oe
	$m = 1.0436$	A1	accept [1.0435, 1.0436]; rejects the negative solution
	4.36	A1	accept 4.36 or [4.359, 4.360]

Question 26 A18 R9 A21 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
26	$1 + \frac{p}{100}$ or $1 - \frac{2p}{100}$	M1	oe, one multiplier, or a trial with a length and width
	$(1 + \frac{p}{100})(1 - \frac{2p}{100}) = 0.9$	M1	oe, e.g. $(100 + p)(100 - 2p) = 9000$
	$p^2 + 50p - 500 = 0$	M1	oe, a three-term quadratic
	$\frac{-50 \pm \sqrt{(50^2 - 4 \times 1 \times -500)}}{2}$	M1	oe, a correct substitution of their values into the formula
	8.54	A1	accept 8.54 or [8.541, 8.542]. Rejects the negative solution