

Proportion

Exam questions

Foundation and Higher tiers

Please write clearly in block capitals.

Name

Class

Teacher

Time allowed: 180 minutes

Materials

For this booklet you need a black pen, a pencil, a ruler, a rubber and a calculator for Section B.

Instructions

- Use black ink or black ballpoint pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer all questions in the spaces provided.
- **Section A** (questions 1 to 24): you must not use a calculator.
- **Section B** (questions 25 to 40): you may use a calculator.
- Do all rough work in this booklet. Cross through any work you do not want marked.

Information

- The marks for each question are shown in brackets.
- The maximum mark for this booklet is 159.

Advice

- Read each question carefully.
- In all calculations, show clearly how you work out your answer.

For teacher’s use

Q	Mark	Q	Mark
1		21	
2		22	
3		23	
4		24	
5		25	
6		26	
7		27	
8		28	
9		29	
10		30	
11		31	
12		32	
13		33	
14		34	
15		35	
16		36	
17		37	
18		38	
19		39	
20		40	
TOTAL			

Section A



You must not use a calculator

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outside
the box

- 1** Here is a recipe for 12 oat biscuits.

Makes 12 oat biscuits	
oats	150 g
butter	90 g
brown sugar	60 g
honey	2 tablespoons

- (a)** Work out how much butter is needed to make 18 oat biscuits.

Answer _____ g

[2 marks]

- (b)** Nia has 400 g of oats.
She has plenty of the other ingredients.

What is the greatest number of oat biscuits she can make?

Answer _____

[2 marks]

- 2 (a) Which ratio is equivalent to 12 : 18?

Circle your answer.

6 : 12 18 : 27 16 : 22 3 : 4

[1 mark]

- (b) $a : 15 = 8 : 6$

Work out the value of a .

Answer _____

[2 marks]

- 3 Here are two tables.

Table A: theatre tickets			
Number of tickets	2	5	8
Total cost (£)	7.00	17.50	28.00

Table B: hiring a van			
Number of hours	2	5	8
Total cost (£)	50.00	95.00	140.00

In one table, the total cost is directly proportional to the number of tickets or the number of hours.

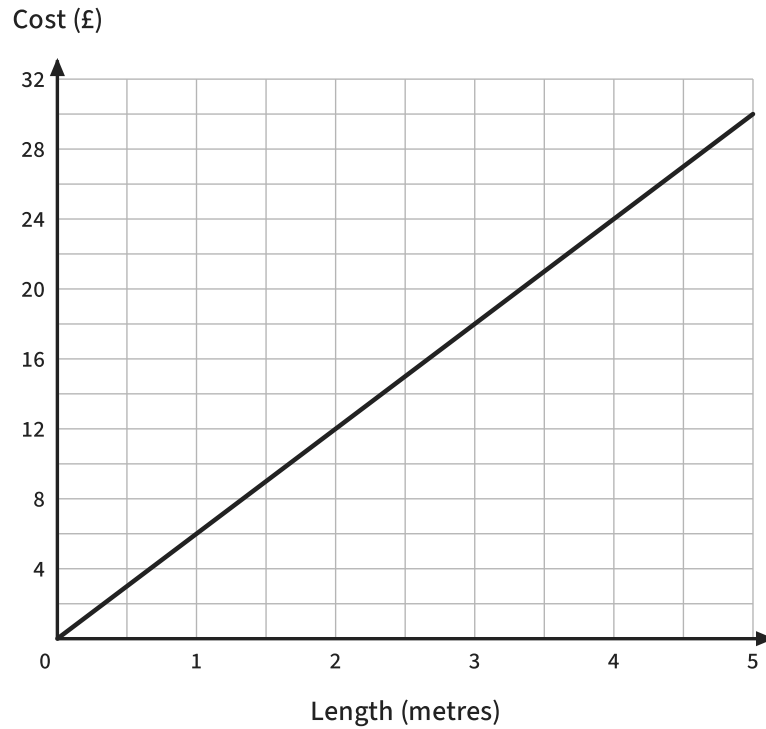
Which table is it?

Give a reason for your answer.

[2 marks]

- 4 The graph shows the cost of lengths of a fabric.

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the box



- (a) Use the graph to find the cost of 3.5 metres of the fabric.

Answer £ _____

[1 mark]

- (b) How does the graph show that the cost is directly proportional to the length?

[1 mark]

- (c) Work out the cost of 18 metres of the fabric.

Answer £ _____

[2 marks]

5 6 volunteers take 5 hours to clear a beach of litter.

- (a) How long would it take 8 volunteers to clear the same beach?
Give your answer in hours and minutes.

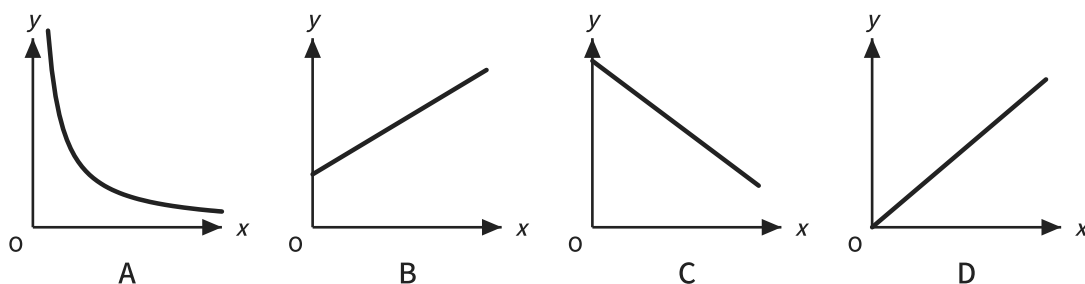
Answer _____

[3 marks]

- (b) Write down one assumption you made.

[1 mark]

6 Here are four graphs.



- (a) Which graph shows that y is directly proportional to x ?

Answer _____

[1 mark]

- (b) Which graph shows that y is inversely proportional to x ?

Answer _____

[1 mark]

- 7** y is inversely proportional to x .

Complete the table.

x	2	5	8	
y	30			4

[3 marks]

- 8** 8 identical pumps can empty a flooded basement in 15 hours
 All 8 pumps start together.
 After 3 hours, 2 of the pumps stop working.
 The other pumps carry on until the basement is empty.
 How long does it take to empty the basement altogether?

Answer _____ hours

[3 marks]

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- 9** The extension, e cm, of a spring is directly proportional to the mass, m grams, hanging from it.

When $m = 150$, $e = 6$

- (a) Work out a formula for e in terms of m .

Answer _____

[2 marks]

- (b) Work out the value of m when $e = 9.2$

Answer _____

[1 mark]

- 10** The brightness, B , of a lamp is inversely proportional to the square of the distance, d metres, from the lamp.

When $d = 2.5$, $B = 64$

- (a) Work out the value of d when $B = 16$

Answer _____

[3 marks]

- (b) Ola says,
"If you move the lamp twice as far away, the brightness halves."
 Is Ola correct?
 Give a reason for your answer.

[1 mark]

11

 y is directly proportional to $(x + 2)^2$ When $x = 1$, $y = 18$ (a) Work out a formula for y in terms of x .

Answer _____

[2 marks]

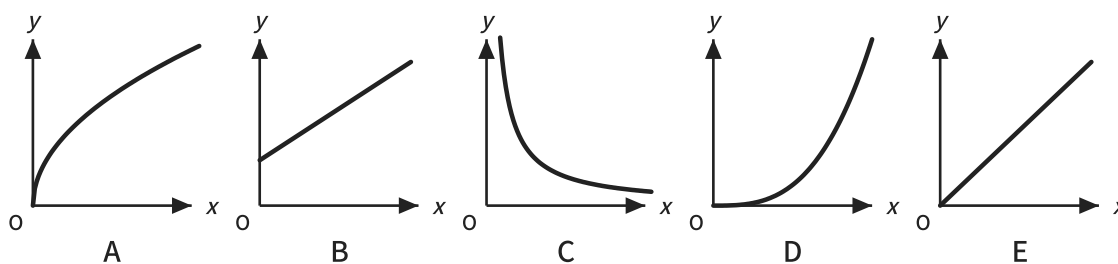
(b) Work out the two values of x when $y = 50$

Answer _____

[2 marks]

12

Here are five sketch graphs.

Each equation below shows a relationship between x and y , for $x > 0$

Write the letter of the graph that could show each relationship.

Equation	$y^2 = 9x$	$xy = 6$	$\frac{y}{x} = 4$	$y = 0.2x^3$
Graph				

[3 marks]

- 13** Here is a table of values for x and y .

x	2	4	5	
y	12	96		768

One of these statements is true.

$$y \propto x \quad y \propto x^2 \quad y \propto x^3$$

- (a) Which statement is true?
You must show your working.

[2 marks]

- (b) Complete the table.

[2 marks]

- 14** y is directly proportional to x .

When $x = a$, $y = 12$

When $x = a + 5$, $y = 27$

Work out the value of a .

Answer _____

[3 marks]

15

Dalia is tiling a floor with square tiles.

The number of tiles needed is inversely proportional to the square of the side length of a tile.

She needs 2400 tiles with side length 15 cm.

Tiles with side length 15 cm cost £3.20 each.

Tiles with side length 20 cm cost £5.50 each.

Which size of tile costs less to cover the floor, and by how much?

You must show your working.

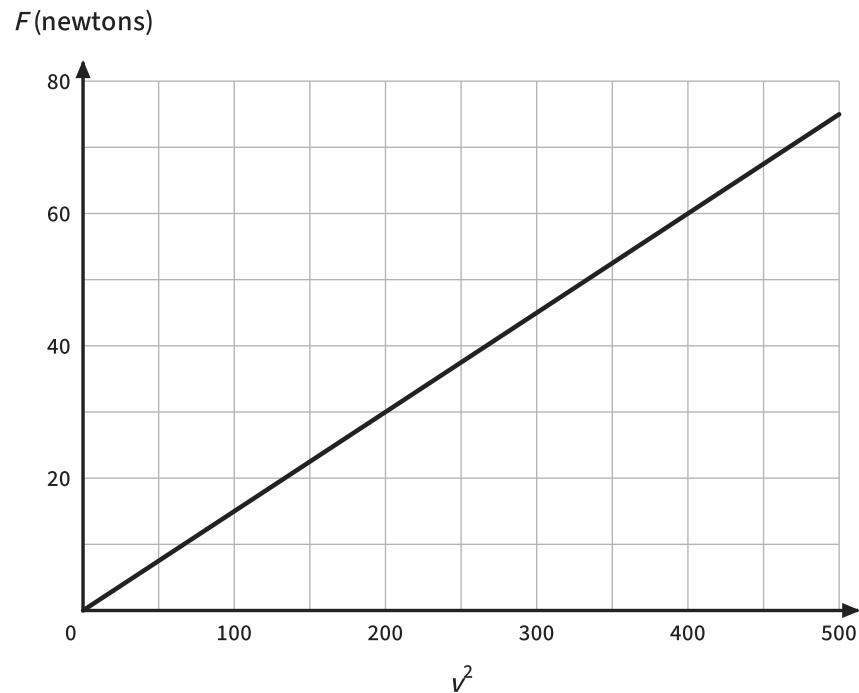
[4 marks]

16

Amir tests a model car in a wind tunnel.

He measures the force, F newtons, on the car at different wind speeds, v m/s.

He plots F against v^2 .



- (a) Explain how the graph shows that F is directly proportional to v^2

[1 mark]

- (b) Work out the force on the car when the wind speed is 30 m/s

Answer _____ N

[2 marks]

- (c) Work out the wind speed when the force is 15 N

Answer _____ m/s

[1 mark]

- 17** A sculptor casts statues in bronze. The statues are all mathematically similar.
The mass of a statue is directly proportional to the cube of its height.
The cost of polishing a statue is directly proportional to the square of its height.

A small statue has a mass of 2 kg and costs £15 to polish.

A large statue has a mass of 54 kg.

Work out the cost of polishing the large statue.

Answer £ _____

[3 marks]

- 18** y is inversely proportional to $\sqrt{x}$
When $x = a$, $y = 6$

(a) Work out the value of y when $x = 4a$

Answer _____

[1 mark]

(b) Work out the value of y when $x = \frac{a}{25}$

Answer _____

[1 mark]

(c) Work out, in terms of a , the value of x when $y = 18$

Answer _____

[2 marks]

19

p is directly proportional to q^2
 q is inversely proportional to $\sqrt{r}$
When $r = 4$, $q = 3$ and $p = 18$

(a) Show that p is inversely proportional to r .

[3 marks]

(b) Work out the value of r when $p = 50$

Answer _____

[2 marks]

20

y is inversely proportional to x^2 , where $x > 0$
When x increases by 2, y decreases by 36%

Work out the value of x before the increase.

Answer _____

[4 marks]

21 y is directly proportional to x^n When $x = 4$, $y = 24$ When $x = 16$, $y = 192$ **(a)** Work out the value of n .

Answer _____

[2 marks]**(b)** Work out the value of y when $x = 36$

Answer _____

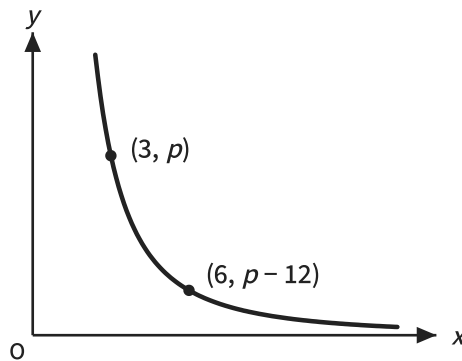
[2 marks]

22

y is inversely proportional to the square of x .

The sketch shows the graph of y against x , for $x > 0$

The graph passes through the points $(3, p)$ and $(6, p - 12)$.



Not drawn accurately

(a) Work out the value of p .

Answer _____

[3 marks]

(b) Work out the value of x when $y = 1$

Answer _____

[1 mark]

23

Two loudspeakers, A and B, are 12 m apart.

The intensity of the sound from a loudspeaker is inversely proportional to the square of the distance from it.

At the same distance, the intensity from A is 9 times the intensity from B.

Point P is on the straight line between A and B.

At P, the intensity of the sound from A is equal to the intensity from B.

Work out the distance AP.

Answer _____ m

[4 marks]

24

A walking club hires a coach for a day trip.

The cost of £720 is shared equally between the people who go.

If 8 more people had gone, each person would have paid £4.50 less.

How many people went on the trip?

You must show your working.

Answer _____

[5 marks]

Section B



You may use a calculator

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25 At a deli counter, 600 g of green olives are priced at £4.62

- (a) Find the price of 420 g of these olives.
Give your answer to the nearest penny.

Answer £ _____

[2 marks]

- (b) The deli scoops olives in amounts that are multiples of 10 g.
Ola has £4 to spend on these olives.
What is the greatest amount she can buy?

Answer _____ g

[2 marks]

26 A garden centre sells bird seed in three sizes of bag.

Small	Medium	Large
750 g £2.85	2 kg £6.90	5 kg £18.25

Which size of bag is the best value for money?
You must show your working.

[3 marks]

- 27** Ewan's car uses 6.2 litres of petrol for every 100 km it travels.
He drives 340 km
Petrol costs 148.9p per litre.

Work out the cost of the petrol he uses.
Give your answer to the nearest penny.

Answer £ _____

[3 marks]

- 28** The speed, v m/s, of a wave in shallow water is directly proportional to the square root of the depth of the water, d metres.
When $d = 2.25$, $v = 4.5$

(a) Work out the speed of the wave when the depth is 0.64 metres.

Answer _____ m/s

[3 marks]

(b) Work out the depth of the water when the speed of the wave is 6.3 m/s

Answer _____ m

[2 marks]

29

The mass of a steel ball is directly proportional to the cube of its radius.

The radius of a second steel ball is 20% greater than the radius of the first ball.

Work out the percentage by which the mass of the second ball is greater than the mass of the first ball.

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Answer _____ %

[3 marks]

30

The time, T seconds, for one complete swing of a pendulum is directly proportional to the square root of its length, L cm.

Bethan times four pendulums. Her results are in the table.

Length, L cm	30	45	72	110
Time, T seconds	1.10	1.34	1.80	2.10

- (a) Bethan recorded one of the times wrongly.
Which length has the wrong time?
You must show your working.

[2 marks]

- (b) Estimate the correct time for the 72 cm pendulum.

Answer _____ seconds

[1 mark]

- (c) Work out the length of a pendulum that takes 2.5 seconds for one complete swing.

Answer _____ cm

[2 marks]

- 31** Lowri melts down a fixed amount of glass to make spherical beads, all the same size. The number of beads she can make is inversely proportional to the cube of the radius of a bead.
She can make 1500 beads with radius 4.5 mm.

How many beads with radius 6 mm can she make?

Answer _____

[3 marks]

- 32** On a camera, the aperture setting, N , controls how wide the lens opens. The exposure time, t seconds, is directly proportional to the square of N .
When $N = 4$, $t = \frac{1}{250}$

Rhiannon wants an exposure time of no more than $\frac{1}{60}$ second.

Her camera lets her choose these aperture settings.

5.6 8 11 16

What is the largest of these aperture settings she can use?

You must show your working.

[4 marks]

- 33** The power, P kilowatts, from a small wind turbine is directly proportional to the cube of the wind speed, v m/s.
When $v = 8$, $P = 1.2$

- (a)** Work out the wind speed that gives a power of 3 kilowatts.
Give your answer to 1 decimal place.

Answer _____ m/s

[3 marks]

- (b)** One afternoon, the wind speed is 6.5 m/s for 2.5 hours and then 11.2 m/s for 1.5 hours.

Energy (kilowatt hours) = power (kilowatts) $\times$ time (hours)

Work out the total energy from the turbine that afternoon.

Give your answer to 1 decimal place.

Answer _____ kWh

[3 marks]

- 34** The strength, S , of the signal a phone receives from a mast is inversely proportional to the square of the distance, d metres, of the phone from the mast.

- (a)** Lena moves her phone so that its distance from the mast increases by 25%
Work out the percentage decrease in the strength of the signal.

Answer _____ %

[2 marks]

- (b)** Jay wants the strength of the signal his phone receives to increase by 50%
Work out the percentage by which he must decrease the distance of his phone from the mast.
Give your answer to 1 decimal place.

Answer _____ %

[3 marks]

35

For each planet, the square of the time it takes to go round the Sun, T days, is directly proportional to the cube of its mean distance from the Sun, d million km.

For the Earth, $T = 365.25$ and $d = 149.6$

(a) For Mars, $d = 227.9$

Work out the value of T for Mars.

Give your answer to the nearest whole number.

Answer _____ days

[3 marks]

(b) For Jupiter, $T = 4333$

Work out the value of d for Jupiter.

Give your answer to 3 significant figures.

Answer _____ million km

[2 marks]

36

The stopping distance of a car is its thinking distance added to its braking distance.

On a test track:

- the thinking distance is directly proportional to the speed, v m/s
- the braking distance is directly proportional to the square of the speed.

At 20 m/s, the thinking distance is 15 m and the braking distance is 32 m.

- (a) Show that the stopping distance, in metres, at a speed of v m/s is $0.75v + 0.08v^2$

[2 marks]

- (b) The car must be able to stop within 85 m.
Work out the greatest speed at which it can travel.
Give your answer to 1 decimal place.

Answer _____ m/s

[4 marks]

37

A tank is filled with water through pipes.

The time taken to fill the tank through one pipe is inversely proportional to the square of the diameter of the pipe.

Pipe A has a diameter of 2.5 cm and fills the tank in 40 minutes.

Pipe B has a diameter of 3.2 cm.

Pipes A and B are turned on together to fill the empty tank.

How long does it take to fill the tank?

Give your answer in minutes and seconds, to the nearest second.

Answer _____

[5 marks]

38

The resistance of a wire is R ohms.

For wires of the same metal and the same diameter, R is directly proportional to the length.

For wires of the same metal and the same length, R is inversely proportional to the square of the diameter.

Wire A is 2.5 m long, has a diameter of 0.8 mm and has a resistance of 0.085 ohms.

- (a) Wire B is made of the same metal as wire A.
Wire B is 4 m long and has a diameter of 0.5 mm.
Work out the resistance of wire B.
Give your answer to 3 significant figures.

Answer _____ ohms

[3 marks]

- (b) Wire C is made of the same metal.
Wire C is 10 m long and has a resistance of 0.2 ohms.
Work out the diameter of wire C.
Give your answer to 2 decimal places.

Answer _____ mm

[2 marks]

- 39** The value of a diamond of a certain quality is directly proportional to the square of its mass.

A diamond of this quality with a mass of 0.7 carats is worth £1850

- (a)** Kai has a diamond of the same quality with a mass of 1.2 carats.
It is cut into two pieces whose masses are in the ratio 2 : 3
Work out the total value lost by cutting the diamond.
Give your answer to the nearest penny.

Answer £ _____

[4 marks]

- (b)** Show that cutting any diamond of this quality into two pieces of equal mass halves its total value.

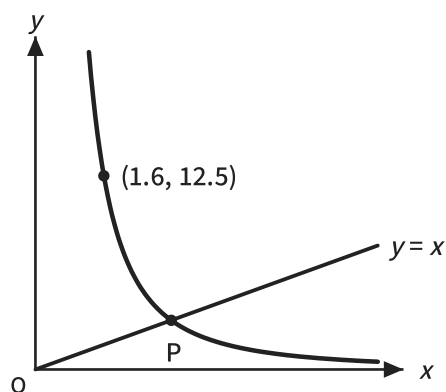
[2 marks]

40

y is inversely proportional to x^2 , where $x > 0$

The sketch shows the line $y = x$ and the graph of y against x , which passes through $(1.6, 12.5)$.

The line and the curve meet at P.



Not drawn accurately

Work out the coordinates of P.

Give your values to 3 significant figures.

P = _____

[4 marks]

END OF QUESTIONS