

Algebraic proof

Exam questions

Foundation and Higher tiers

Please write clearly in block capitals.

Name

Class

Teacher

Time allowed: 55 minutes

Materials

For this booklet you need a black pen, a pencil, a ruler and a rubber.

Instructions

- Use black ink or black ballpoint pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer all questions in the spaces provided.
- You must not use a calculator.
- Do all rough work in this booklet. Cross through any work you do not want marked.

Information

- The marks for each question are shown in brackets.
- The maximum mark for this booklet is 50.

Advice

- Read each question carefully.
- In all calculations, show clearly how you work out your answer.

For teacher's use

Question	Mark
1	
2	
3	
4	
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11	
12	
13	
14	
15	
16	
TOTAL	

- 1** Tick one box in each row to show whether it is an equation, an identity, an expression or a formula.

	Equation	Identity	Expression	Formula
$5x + 2 = 17$				
$3(x + 2) \equiv 3x + 6$				
$P = 2l + 2w$				
$4x - 7$				

[2 marks]

- 2** Tom says,
“When n is an odd number, $n^2 + 4$ is always a prime number.”

Show that Tom is wrong.

[2 marks]

- 3** Kat says,
“ $(x + 3)^2 \equiv x^2 + 9$ ”

Show that Kat is wrong.

[2 marks]

4 Show that $(x+5)^2 - (x+1)^2 \equiv 8(x+3)$

[3 marks]

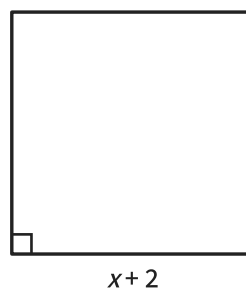
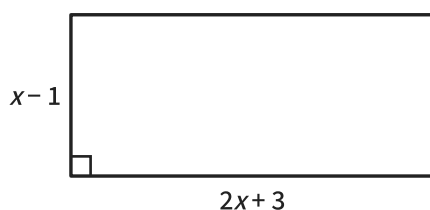
5 $4(2x-3) - 2(x-a) \equiv bx+2$

Work out the values of a and b .

Answer _____

[3 marks]

- 6 Here is a rectangle and a square.
All lengths are in centimetres.



Not drawn accurately

Show that the area of the rectangle minus the area of the square is $x^2 - 3x - 7$

[3 marks]

- 7** Prove that the sum of any four consecutive integers is always 2 more than a multiple of 4

[3 marks]

- 8** m is an even number and n is an odd number.

Prove that $m^2 + n^2$ is always 1 more than a multiple of 4

[3 marks]

- 9** n is an integer.

Prove that $(n + 5)(n + 1) + 4$ is always a square number.

[3 marks]

- 10** Sam is asked to prove that the sum of two consecutive even numbers is never a multiple of 4

Here is his working.

Let the two numbers be $2n$ and $2m$

$$2n + 2m = 2(n + m)$$

This is a multiple of 2, so it is not a multiple of 4

- (a)** Write down one mistake that Sam has made.

[1 mark]

- (b)** Write a correct proof.

[3 marks]

- 11** a and b are both multiples of 5

Prove that $a^2 - b^2$ is a multiple of 25

[3 marks]

- 12** Prove that one more than the product of two consecutive even numbers is always the square of an odd number.

[4 marks]

- 13** n is a positive number.

Prove that $\frac{n}{n+1} - \frac{n-1}{n} \equiv \frac{1}{n(n+1)}$

[3 marks]

- 14** n is an integer greater than 1

Prove that $n^3 - n$ is always a multiple of 6

[4 marks]

- 15** x and y are positive numbers.
 $x + y = 10$

Prove that $xy \leq 25$

[4 marks]

- 16** x and y are different positive numbers.

Prove that $\frac{x}{y} + \frac{y}{x} > 2$

[4 marks]

END OF QUESTIONS