

Algebraic proof

Foundation and Higher tiers | Total marks: 50

Marks

- M** Method mark, for a correct method. It can be implied by a correct answer.
- A** Accuracy mark, for a correct answer. It needs the method mark before it.
- B** A mark for a correct value or statement, with or without working.
- ft** Follow through: the mark is given for using an earlier wrong answer correctly.
- oe** Or equivalent.
- [a, b]** Any value from a to b inclusive.

Questions

Q	Skill	Spec	Tier	Grade	Calculator	Marks
1	Equations, identities, expressions and formulae	A3 A6	Common	3	No	2
2	Counterexamples	A6 N4	Common	4	No	2
3	Showing something is not an identity	A6 A4	Common	4	No	2
4	Showing an identity	A6 A4	Common	5	No	3
5	Unknown coefficients in an identity	A6 A4	Common	5	No	3
6	Showing an expression for an area	A6 A4	Common	5	No	3
7	Proof with consecutive integers	A6	Higher	6	No	3
8	Proof with odd and even numbers	A6	Higher	6	No	3
9	Proof that an expression is a square	A6 A4	Higher	7	No	3
10	Spot the error in a proof	A6	Higher	7	No	4
11	Proof with multiples	A6	Higher	7	No	3
12	Proof with squares	A6	Higher	8	No	4
13	Proof with algebraic fractions	A6 A4	Higher	8	No	3
14	Proof by factorising: a product of consecutive integers	A6 A4	Higher	8	No	4
15	Proof by completing the square	A6 A18	Higher	9	No	4
16	Proof with algebraic fractions and a square	A6 A4	Higher	9	No	4
	Total					50

Question 1 A3 A6 · Common · Grade 3 · 2 marks

Q	Answer	Mark	Comments
1	$5x + 2 = 17$: equation; $3(x + 2) \equiv 3x + 6$: identity; $P = 2l + 2w$: formula; $4x - 7$: expression	B2	B1 for two or three correct

Question 2 A6 N4 · Common · Grade 4 · 2 marks

Q	Answer	Mark	Comments
2	substitutes an odd value of n that gives a number that is not prime	M1	e.g. $n = 9$ gives 85 or $n = 11$ gives 125
	$85 = 5 \times 17$ or $125 = 5 \times 25$, so it is not prime	A1	oe, a correct counterexample with a reason

Question 3 A6 A4 · Common · Grade 4 · 2 marks

Q	Answer	Mark	Comments
3	$(x + 3)(x + 3) = x^2 + 3x + 3x + 9$	M1	or substitutes a value other than 0, e.g. $x = 1$ gives 16 and 10
	$x^2 + 6x + 9$, which is not $x^2 + 9$	A1	or $16 \neq 10$ with a conclusion

Question 4 A6 A4 · Common · Grade 5 · 3 marks

Q	Answer	Mark	Comments
4	$x^2 + 10x + 25$ or $x^2 + 2x + 1$	M1	one correct expansion
	$x^2 + 10x + 25 - x^2 - 2x - 1$	M1	oe, both expansions with the bracket subtracted correctly
	$8x + 24 = 8(x + 3)$	A1	a full method with no errors

Question 5 A6 A4 · Common · Grade 5 · 3 marks

Q	Answer	Mark	Comments
5	$8x - 12 - 2x + 2a$	M1	oe, a correct expansion
	$b = 6$	A1	
	$a = 7$	A1	from $-12 + 2a = 2$

Question 6 A6 A4 · Common · Grade 5 · 3 marks

Q	Answer	Mark	Comments
6	$(2x + 3)(x - 1) = 2x^2 + x - 3$	M1	oe, the area of the rectangle
	$(x + 2)^2 = x^2 + 4x + 4$	M1	oe, the area of the square
	$2x^2 + x - 3 - x^2 - 4x - 4 = x^2 - 3x - 7$	A1	a full method with the subtraction shown

Question 7 A6 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
7	$n + n + 1 + n + 2 + n + 3$	M1	oe, four consecutive integers in terms of one letter
	$4n + 6$	M1	oe
	$4n + 6 = 4(n + 1) + 2$, which is 2 more than a multiple of 4	A1	oe, e.g. $4n + 4 + 2$ with a conclusion

Question 8 A6 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
8	$m = 2a$ and $n = 2b + 1$	M1	oe, two different letters
	$4a^2 + 4b^2 + 4b + 1$	M1	oe, both squares expanded correctly
	$4(a^2 + b^2 + b) + 1$, which is 1 more than a multiple of 4	A1	a conclusion is needed

Question 9 A6 A4 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
9	$n^2 + 6n + 5 + 4$	M1	oe, a correct expansion
	$n^2 + 6n + 9 = (n + 3)^2$	M1	factorises correctly
	$(n + 3)^2$ with a statement that this is the square of an integer	A1	a conclusion is needed

Question 10 A6 · Higher · Grade 7 · 4 marks

Q	Answer	Mark	Comments
10(a)	$2n$ and $2m$ do not have to be consecutive, the next even number is $2n + 2$	B1	oe, or a multiple of 2 can also be a multiple of 4
10(b)	$2n + 2n + 2$	M1	oe, two consecutive even numbers in terms of one letter
	$4n + 2$	M1	
	$4n + 2$ is 2 more than a multiple of 4, so it is never a multiple of 4	A1	oe, a conclusion is needed

Question 11 A6 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
11	$a = 5m$ and $b = 5k$	M1	oe, two different letters
	$25m^2 - 25k^2$	M1	
	$25(m^2 - k^2)$, which is a multiple of 25	A1	a conclusion is needed

Question 12 A6 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
12	$2n(2n + 2) + 1$	M1	oe
	$4n^2 + 4n + 1$	M1	
	$(2n + 1)^2$	M1	factorises correctly
	$2n + 1$ is odd, so it is the square of an odd number	A1	a conclusion is needed

Question 13 A6 A4 · Higher · Grade 8 · 3 marks

Q	Answer	Mark	Comments
13	$\frac{n \times n - (n - 1)(n + 1)}{n(n + 1)}$	M1	oe, a common denominator
	$n^2 - (n^2 - 1)$	M1	oe, the numerator expanded correctly
	$\frac{1}{n(n + 1)}$	A1	a full method with no errors

Question 14 A6 A4 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
14	$n(n^2 - 1)$	M1	a common factor taken out
	$(n - 1)n(n + 1)$	M1	oe, fully factorised
	these are three consecutive integers, so at least one of them is even and one of them is a multiple of 3	M1	oe
	so the product is a multiple of $2 \times 3 = 6$	A1	a full proof with a conclusion. Trying values of n earns no marks

Question 15 A6 A18 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
15	$x(10 - x)$ or $10x - x^2$	M1	oe, xy in terms of one letter
	$-(x^2 - 10x)$	M1	oe, a first step to completing the square
	$25 - (x - 5)^2$	A1	oe
	$(x - 5)^2 \geq 0$ so $25 - (x - 5)^2 \leq 25$	A1	a full reason

Question 16 A6 A4 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
16	$\frac{x^2+y^2}{xy}$	M1	oe, a single fraction
	$\frac{x^2+y^2}{xy} - 2 = \frac{x^2-2xy+y^2}{xy}$	M1	oe, or $x^2+y^2 > 2xy$ to be shown, using $xy > 0$
	$\frac{(x-y)^2}{xy}$	M1	oe, or $(x-y)^2 > 0$
	$(x-y)^2 > 0$ because $x \neq y$, and $xy > 0$, so $\frac{x}{y} + \frac{y}{x} - 2 > 0$	A1	a full proof with a conclusion; both reasons needed