

# Powers, roots and indices

Foundation and Higher tiers | Total marks: 74

## Marks

- M** Method mark, for a correct method. It can be implied by a correct answer.
- A** Accuracy mark, for a correct answer. It needs the method mark before it.
- B** A mark for a correct value or statement, with or without working.
- ft** Follow through: the mark is given for using an earlier wrong answer correctly.
- oe** Or equivalent.
- [a, b]** Any value from a to b inclusive.

## Questions

| Q  | Skill                                                | Spec   | Tier       | Grade | Calculator | Marks |
|----|------------------------------------------------------|--------|------------|-------|------------|-------|
| 1  | Recognising squares, cubes and powers of 2           | N6     | Foundation | 2     | No         | 3     |
| 2  | Powers and roots in a calculation                    | N6     | Foundation | 3     | No         | 2     |
| 3  | Laws of indices with numbers                         | N7     | Common     | 4     | No         | 3     |
| 4  | Square roots in context                              | N6     | Foundation | 4     | No         | 3     |
| 5  | Zero and negative indices                            | N7     | Common     | 5     | No         | 3     |
| 6  | Finding an unknown index                             | N7     | Common     | 5     | No         | 4     |
| 7  | Index laws: same base                                | N7     | Common     | 5     | No         | 3     |
| 8  | Estimating a square root                             | N6     | Higher     | 6     | No         | 3     |
| 9  | Fractional indices                                   | N7     | Higher     | 6     | No         | 3     |
| 10 | Ordering powers with negative and fractional indices | N7     | Higher     | 7     | No         | 2     |
| 11 | Solving an index equation                            | N7     | Higher     | 8     | No         | 3     |
| 12 | Negative fractional indices: show that               | N7     | Higher     | 8     | No         | 3     |
| 13 | Comparing powers with index laws                     | N7     | Higher     | 9     | No         | 3     |
| 14 | Simultaneous equations with indices                  | N7 A19 | Higher     | 9     | No         | 4     |
| 15 | Testing a claim about a square and a cube            | N6     | Foundation | 2     | Yes        | 3     |
| 16 | Powers and roots on a calculator                     | N6 N15 | Foundation | 4     | Yes        | 3     |

| Q  | Skill                                                 | Spec      | Tier       | Grade | Calculator | Marks     |
|----|-------------------------------------------------------|-----------|------------|-------|------------|-----------|
| 17 | Cube roots: volume in context                         | N6 G16    | Foundation | 5     | Yes        | 3         |
| 18 | Powers in a growth formula                            | N6 R16 N9 | Foundation | 6     | Yes        | 4         |
| 19 | Negative and fractional indices on a calculator       | N7 N6     | Higher     | 6     | Yes        | 4         |
| 20 | A formula with a fractional index, used both ways     | N7 A5     | Higher     | 7     | Yes        | 5         |
| 21 | Proportion through a fractional index                 | N7 R13    | Higher     | 8     | Yes        | 3         |
| 22 | Equations with fractional indices: how many solutions | N7        | Higher     | 9     | Yes        | 3         |
| 23 | Percentage change through a fractional index          | N7 R9     | Higher     | 9     | Yes        | 4         |
|    | <b>Total</b>                                          |           |            |       |            | <b>74</b> |

**Question 1** N6 · Foundation · Grade 2 · 3 marks

| Q    | Answer    | Mark | Comments                |
|------|-----------|------|-------------------------|
| 1(a) | 8, 27, 64 | B1   | all three and no others |
| 1(b) | 64        | B1   |                         |
| 1(c) | 8 or 32   | B1   |                         |

**Question 2** N6 · Foundation · Grade 3 · 2 marks

| Q | Answer         | Mark | Comments            |
|---|----------------|------|---------------------|
| 2 | 81 or 32 or 49 | M1   | one of them correct |
|   | 7              | A1   |                     |

**Question 3** N7 · Common · Grade 4 · 3 marks

| Q    | Answer   | Mark | Comments |
|------|----------|------|----------|
| 3(a) | $5^7$    | B1   |          |
| 3(b) | $7^6$    | B1   |          |
| 3(c) | $2^{12}$ | B1   |          |

**Question 4** N6 · Foundation · Grade 4 · 3 marks

| Q | Answer                                           | Mark | Comments                      |
|---|--------------------------------------------------|------|-------------------------------|
| 4 | $\sqrt{3600}$ or 60                              | M1   | oe, or $\sqrt{90\,000} = 300$ |
|   | $4 \times 5 \times 60$ or $4 \times 300$ or 1200 | M1   | oe, the perimeter in cm       |
|   | 12                                               | A1   |                               |

**Question 5** N7 · Common · Grade 5 · 3 marks

| Q    | Answer        | Mark | Comments  |
|------|---------------|------|-----------|
| 5(a) | 1             | B1   |           |
| 5(b) | $\frac{1}{8}$ | B1   | not 0.125 |
| 5(c) | $5^{-2}$      | B1   |           |

### Question 6 N7 · Common · Grade 5 · 4 marks

| Q    | Answer        | Mark | Comments                     |
|------|---------------|------|------------------------------|
| 6(a) | $2n + 3 = 11$ | M1   | oe                           |
|      | 4             | A1   |                              |
| 6(b) | $8 = 2^3$     | M1   | oe, or $2^{10} \div 8 = 128$ |
|      | 7             | A1   |                              |

### Question 7 N7 · Common · Grade 5 · 3 marks

| Q    | Answer                                               | Mark | Comments                                                      |
|------|------------------------------------------------------|------|---------------------------------------------------------------|
| 7(a) | the index laws only work when the bases are the same | B1   | oe, or works out $81 \times 8 = 648$ and says it is not $6^7$ |
| 7(b) | $4^2 = 2^4$ or 16                                    | M1   | oe                                                            |
|      | $2^9$                                                | A1   |                                                               |

### Question 8 N6 · Higher · Grade 6 · 3 marks

| Q    | Answer                              | Mark | Comments                                                                  |
|------|-------------------------------------|------|---------------------------------------------------------------------------|
| 8(a) | 9 and 10                            | B1   |                                                                           |
| 8(b) | $9.1^2 = 82.81$ and $9.2^2 = 84.64$ | M1   | oe, trials either side of 83, or interpolation such as $9 + \frac{2}{19}$ |
|      | 9.1                                 | A1   |                                                                           |

### Question 9 N7 · Higher · Grade 6 · 3 marks

| Q    | Answer                | Mark | Comments              |
|------|-----------------------|------|-----------------------|
| 9(a) | 6                     | B1   |                       |
| 9(b) | $\sqrt[3]{1000} = 10$ | M1   | oe, or 1 000 000 seen |
|      | 100                   | A1   |                       |

### Question 10 N7 · Higher · Grade 7 · 2 marks

| Q  | Answer                                                                            | Mark | Comments                                                                                                |
|----|-----------------------------------------------------------------------------------|------|---------------------------------------------------------------------------------------------------------|
| 10 | $3^{-2}, 16^{-\frac{1}{4}}, 5^0, 27^{\frac{1}{3}}, \left(\frac{1}{2}\right)^{-2}$ | B2   | B1 for four of the values correct ( $\frac{1}{9}$ , 3, 1, $\frac{1}{2}$ , 4) or one number out of place |

**Question 11** N7 · Higher · Grade 8 · 3 marks

| Q  | Answer                                                         | Mark | Comments                               |
|----|----------------------------------------------------------------|------|----------------------------------------|
| 11 | two of $25 = 5^2$ , $125 = 5^3$ , $\sqrt{5} = 5^{\frac{1}{2}}$ | M1   | oe                                     |
|    | $2x = 3 - \frac{1}{2}$                                         | M1   | oe, such as $5^{2x} = 5^{\frac{5}{2}}$ |
|    | $\frac{5}{4}$                                                  | A1   | oe, such as 1.25                       |

**Question 12** N7 · Higher · Grade 8 · 3 marks

| Q  | Answer                                                                                           | Mark | Comments                                     |
|----|--------------------------------------------------------------------------------------------------|------|----------------------------------------------|
| 12 | $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ or $\sqrt[3]{\left(\frac{8}{27}\right)} = \frac{2}{3}$ | M1   | oe, deals with the negative or the cube root |
|    | $\frac{9}{4}$ and $16^{\frac{1}{4}} = 2$                                                         | M1   | oe, or $\frac{4}{9}$ and 2                   |
|    | $\frac{9}{4} \times 2 = \frac{9}{2}$                                                             | A1   | a full method with no errors                 |

**Question 13** N7 · Higher · Grade 9 · 3 marks

| Q  | Answer                                                     | Mark | Comments                                                      |
|----|------------------------------------------------------------|------|---------------------------------------------------------------|
| 13 | $2^{160} = (2^8)^{20}$ or $256^{20}$                       | M1   | oe, a common power such as 20, or a base such as $(2^{32})^5$ |
|    | $3^{100} = (3^5)^{20}$ or $243^{20}$                       | M1   | oe, with the same power                                       |
|    | <b>256 &gt; 243 so <math>256^{20} &gt; 243^{20}</math></b> | A1   | a full argument                                               |

**Question 14** N7 A19 · Higher · Grade 9 · 4 marks

| Q  | Answer                                           | Mark | Comments                       |
|----|--------------------------------------------------|------|--------------------------------|
| 14 | $2^{3x+2y} = 2^7$ or $3x + 2y = 7$               | M1   | oe                             |
|    | $3^{3x-2y} = 3^{-1}$ or $3x - 2y = -1$           | M1   | oe                             |
|    | a correct method to solve their two equations    | M1   | such as adding to get $6x = 6$ |
|    | <b><math>x = 1</math> and <math>y = 2</math></b> | A1   |                                |

**Question 15** N6 · Foundation · Grade 2 · 3 marks

| Q     | Answer                           | Mark | Comments              |
|-------|----------------------------------|------|-----------------------|
| 15(a) | 17.64 or 17.576                  | M1   | one of the two values |
|       | <b>Yes with 17.64 and 17.576</b> | A1   |                       |
| 15(b) | <b>6.075</b>                     | B1   |                       |

### Question 16 N6 N15 · Foundation · Grade 4 · 3 marks

| Q     | Answer              | Mark | Comments                         |
|-------|---------------------|------|----------------------------------|
| 16(a) | 243.444 or 15.60... | M1   | 6.25 alone is not enough         |
|       | 2.4964307...        | A1   |                                  |
| 16(b) | 2.50                | B1ft | must show the zero; ft their (a) |

### Question 17 N6 G16 · Foundation · Grade 5 · 3 marks

| Q  | Answer                                               | Mark | Comments                      |
|----|------------------------------------------------------|------|-------------------------------|
| 17 | $3.2^3$ or 32.768                                    | M1   |                               |
|    | $\sqrt[3]{(50 \times 32.768)}$ or $\sqrt[3]{1638.4}$ | M1   | oe, ft their 32.768           |
|    | 11.8                                                 | A1   | accept 11.8 or [11.78, 11.79] |

### Question 18 N6 R16 N9 · Foundation · Grade 6 · 4 marks

| Q     | Answer                                                                | Mark | Comments                        |
|-------|-----------------------------------------------------------------------|------|---------------------------------|
| 18(a) | $9.8415 \times 10^5$                                                  | B1   | 984 150 in ordinary form is B0  |
| 18(b) | a correct trial, such as $50 \times 3^{10} = 2\,952\,450$             | M1   | oe                              |
|       | $50 \times 3^{10} = 2\,952\,450$ and $50 \times 3^{11} = 8\,857\,350$ | M1   | trials either side of 5 000 000 |
|       | 11                                                                    | A1   |                                 |

### Question 19 N7 N6 · Higher · Grade 6 · 4 marks

| Q     | Answer                                                       | Mark | Comments                                                                            |
|-------|--------------------------------------------------------------|------|-------------------------------------------------------------------------------------|
| 19(a) | $148.877 - 6.449...$ or $142.42...$ or $1.7^{-2} = 0.346...$ | M1   | oe                                                                                  |
|       | 412                                                          | A1   | accept 411.61...                                                                    |
| 19(b) | he worked out $64^1 \div 3$                                  | B1   | oe, such as he needed brackets round $\frac{1}{3}$ or he did not take the cube root |
|       | 4                                                            | B1   |                                                                                     |

### Question 20 N7 A5 · Higher · Grade 7 · 5 marks

| Q     | Answer                                                         | Mark | Comments           |
|-------|----------------------------------------------------------------|------|--------------------|
| 20(a) | $70 \times 520^{\frac{3}{4}}$                                  | M1   | oe, or 108.8...    |
|       | 7620                                                           | A1   | accept 7622.5...   |
| 20(b) | $m^{\frac{3}{4}} = 15$                                         | M1   | oe, $1050 \div 70$ |
|       | $15^{\frac{4}{3}}$ or $(\sqrt[3]{15})^4$ or $\sqrt[3]{(15^4)}$ | M1   | oe                 |
|       | 37                                                             | A1   | accept 36.99...    |

**Question 21** N7 R13 · Higher · Grade 8 · 3 marks

| Q         | Answer                | Mark      | Comments                                                                 |
|-----------|-----------------------|-----------|--------------------------------------------------------------------------|
| <b>21</b> | $a = 7^{\frac{2}{3}}$ | <b>M1</b> | oe, or a pair of times such as 1 and 3.5 with their distances            |
|           | $3.5^{\frac{2}{3}}$   | <b>M1</b> | oe, or distances found from chosen times such as 1 and 3.5, then divided |
|           | <b>2.31</b>           | <b>A1</b> | accept 2.3052...                                                         |

**Question 22** N7 · Higher · Grade 9 · 3 marks

| Q            | Answer                                  | Mark      | Comments                                                                             |
|--------------|-----------------------------------------|-----------|--------------------------------------------------------------------------------------|
| <b>22(a)</b> | $7.2^{\frac{3}{2}}$ or $(\sqrt{7.2})^3$ | <b>M1</b> | oe                                                                                   |
|              | <b>19.3 and -19.3</b>                   | <b>A1</b> | both needed                                                                          |
| <b>22(b)</b> | <b>No, with a reason</b>                | <b>B1</b> | such as it means $(\sqrt{x})^3$ and $\sqrt{x}$ needs $x \geq 0$ , so only $x = 3.73$ |

**Question 23** N7 R9 · Higher · Grade 9 · 4 marks

| Q            | Answer                          | Mark      | Comments                                                               |
|--------------|---------------------------------|-----------|------------------------------------------------------------------------|
| <b>23(a)</b> | $1.2^{\frac{5}{2}}$ or 1.577... | <b>M1</b> | oe, or a worked example with values                                    |
|              | <b>57.7</b>                     | <b>A1</b> | accept 57.7 or [57.74, 57.75]                                          |
| <b>23(b)</b> | $2^{\frac{2}{5}}$ or 1.319...   | <b>M1</b> | oe, such as the 5th root of 4, or trials such as $1.3^{2.5} = 1.92...$ |
|              | <b>32.0</b>                     | <b>A1</b> | accept 32.0 or [31.95, 31.96]                                          |