

Functions and iteration

Exam questions

Foundation and Higher tiers

Please write clearly in block capitals.

Name

Class

Teacher

Time allowed: 100 minutes

Materials

For this booklet you need a black pen, a pencil, a ruler, a rubber and a calculator for Section B.

Instructions

- Use black ink or black ballpoint pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer all questions in the spaces provided.
- **Section A** (questions 1 to 12): you must not use a calculator.
- **Section B** (questions 13 to 21): you may use a calculator.
- Do all rough work in this booklet. Cross through any work you do not want marked.

Information

- The marks for each question are shown in brackets.
- The maximum mark for this booklet is 89.

Advice

- Read each question carefully.
- In all calculations, show clearly how you work out your answer.

For teacher's use

Q	Mark	Q	Mark
1		12	
2		13	
3		14	
4		15	
5		16	
6		17	
7		18	
8		19	
9		20	
10		21	
11			
TOTAL			

Section A



You must not use a calculator

Do not
write
outside
the box

- 1 Here is a function machine.



- (a) Work out the output when the input is 7

Answer _____

[1 mark]

- (b) Work out the input when the output is 22

Answer _____

[2 marks]

- 2 Here is a function machine.



Complete the table.

Input	Output
10	
	7
-2	

[3 marks]

- 3** Here is a function machine.



- (a) The input is n .
Write an expression, in terms of n , for the output.

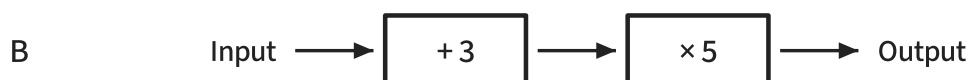
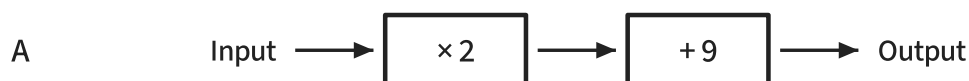
Answer _____

[1 mark]

- (b) The input is a whole number.
Explain why the output is always an odd number.

[2 marks]

- 4** Here are two function machines, **A** and **B**.



The same number is put into both machines.
The two outputs are the same.
Work out the number that is put in.

Answer _____

[3 marks]

5 $f(x) = 3x^2 - 5$

(a) Work out $f(-2)$

Answer _____

[1 mark]

(b) Solve $f(x) = 70$

Answer _____

[2 marks]

6 $f(x) = \frac{2x-1}{5}$

(a) Find $f^{-1}(x)$

Answer _____

[2 marks]

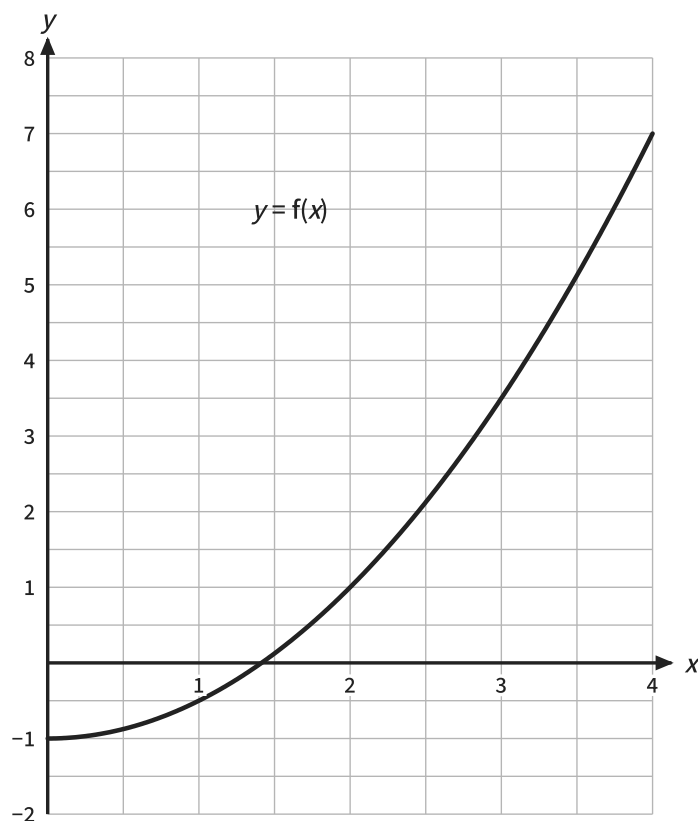
(b) Work out $f^{-1}(3)$

Answer _____

[1 mark]

- 7 The graph of $y = f(x)$ is drawn for values of x from 0 to 4

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the box



- (a) Use the graph to find $f(3)$

Answer _____

[1 mark]

- (b) Use the graph to find $f^{-1}(1)$

Answer _____

[1 mark]

- (c) Use the graph to find $ff(2)$

Answer _____

[1 mark]

8 $f(x) = x + 4$
 $g(x) = x^2$

(a) Work out $fg(3)$

Answer _____
[1 mark]

(b) Write down an expression for $gf(x)$

Answer _____
[1 mark]

(c) Solve $fg(x) = gf(x)$

Answer _____
[2 marks]

9 $f(x) = 2x + 1$
 $fg(x) = 6x - 7$

Find $g(x)$

Answer _____
[2 marks]

10 $f(x) = \frac{x+6}{x-1}$ for $x \neq 1$

Show that $f^{-1}(x) = f(x)$

[3 marks]

11 $f(x) = x^2 - 6x$
 $g(x) = x + k$ where k is a constant
 The graph of $y = fg(x)$ has a turning point where $x = -2$

(a) Work out the value of k .

$k =$ _____

[3 marks]

(b) Work out the coordinates of the turning point of the graph of $y = fg(x)$

Answer _____

[2 marks]

12

$$f(x) = \frac{1}{x} \text{ for } x \neq 0$$

$$g(x) = 2x - 1$$

Show that the equation $fg(x) = gf(x)$ has exactly one solution, and find it.

Do not
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outside
the box

$x =$ _____

[4 marks]

Section B

You may use a calculator

Do not
write
outside
the box

- 13** Here is an iteration formula.

$$x_{n+1} = 4 - \frac{1}{x_n}$$

- (a)** Starting with $x_0 = 3$, work out the values of x_1 , x_2 and x_3

Answer _____

[2 marks]

- (b)** The values of x_n get closer and closer to a number α .

Show that α is a solution of $x^2 - 4x + 1 = 0$

[2 marks]

- 14** (a) Show that the equation $x^3 - 5x + 1 = 0$ has a solution between $x = 2$ and $x = 3$

[2 marks]

- (b) Show that $x^3 - 5x + 1 = 0$ can be rearranged to give $x = \sqrt[3]{5x - 1}$

[1 mark]

- (c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt[3]{5x_n - 1}$ to work out the values of x_1 , x_2 and x_3

Answer _____

[2 marks]

- (d) Mo says,
" x_3 rounded to 2 decimal places gives a solution of $x^3 - 5x + 1 = 0$ correct to 2 decimal places."
 Is Mo correct?
 You must show your working.

[2 marks]

- 15** The function f changes a temperature in degrees Celsius to degrees Fahrenheit.

$$f(x) = 1.8x + 32$$

The function g changes a temperature in degrees Celsius to kelvin.

$$g(x) = x + 273.15$$

- (a)** Find $g^{-1}(x)$

Answer _____

[1 mark]

- (b)** Show that $fg^{-1}(x) = 1.8x - 459.67$

[2 marks]

- (c)** There is one temperature at which the number of kelvin is the same as the number of degrees Fahrenheit.
Work out this temperature in kelvin.

Answer _____ K

[2 marks]

16

Duckweed is a plant that grows on the surface of a pond.

A_n is the area, in m^2 , covered by duckweed n weeks after it was first seen.

The area is modelled by

$$A_{n+1} = 1.4A_n - 0.008A_n^2$$

where $A_0 = 5$

- (a) Work out the values of A_1 , A_2 and A_3

Answer _____

[2 marks]

- (b) The values of A_n get closer and closer to a number L .
Work out the value of L .

$L =$ _____

[2 marks]

- (c) The pond has an area of 60 m^2 .
Does the model show that duckweed will cover the whole pond?
Give a reason for your answer.

[1 mark]

17

The equation $x^3 - 6x + 2 = 0$ has a solution between $x = 2$ and $x = 3$
Here are two iteration formulae made by rearranging the equation.

A $x_{n+1} = \frac{x_n^3 + 2}{6}$

B $x_{n+1} = \sqrt[3]{6x_n - 2}$

(a) Lara uses formula **A** with $x_0 = 2$

Work out x_1 , x_2 and x_3 , and explain why formula **A** will not help Lara to find the solution between 2 and 3

[2 marks]

(b) Use formula **B** with $x_0 = 2$ to find the solution between 2 and 3, correct to 2 decimal places.

You must show that your answer is correct to 2 decimal places.

Answer _____

[3 marks]

- 18** A paint tin is a cylinder with radius x cm.
The height of the tin is 4 cm more than its radius.
The volume of the tin is 500 cm^3

(a) Show that $x = \sqrt{\frac{500}{\pi(x+4)}}$

[2 marks]

- (b) Starting with $x_0 = 4$, use the iteration formula $x_{n+1} = \sqrt{\frac{500}{\pi(x_n+4)}}$ to find the height of the tin.
Give your answer correct to 1 decimal place.

Answer _____ cm

[2 marks]

- (c) Show that your answer to part (b) is correct to 1 decimal place.

[2 marks]

- 19** $f(x) = ax + b$ where a and b are constants and $a > 0$
 $ff(x) = 1.8225x + 5.64$

(a) Work out the values of a and b

Answer _____

[4 marks]

- (b)** Solve $f(x) = f^{-1}(x)$
Give your answer correct to 3 significant figures.

Answer _____

[2 marks]

20

Ellie pays £2500 into a savings account at the start of each year for 3 years.

The account pays compound interest at $r\%$ per year.

At the end of the 3 years there is £8100 in the account.

The multiplier for the interest is m , where $m = 1 + \frac{r}{100}$

(a) Show that $m^3 + m^2 + m = 3.24$

[2 marks]

(b) Show that this equation can be rearranged to give $m = \sqrt{\frac{3.24 - m}{m + 1}}$

[1 mark]

(c) Starting with $m_0 = 1$, use the iteration formula $m_{n+1} = \sqrt{\frac{3.24 - m_n}{m_n + 1}}$ to find the value of r .
Give your answer correct to 1 decimal place.

$r =$ _____

[3 marks]

21

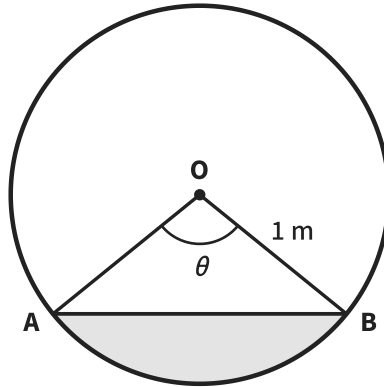
A tank is a cylinder lying on its side.

The diagram shows the circular cross-section of the tank, centre O, radius 1 m.

The oil in the tank fills the shaded segment. AB is the surface of the oil.

Angle AOB = θ°

The area of the shaded segment is 0.4 m^2



- (a) Show that $\theta = \frac{180}{\pi} (0.8 + \sin \theta)$

[2 marks]

- (b) Starting with $\theta_0 = 100$, use the iteration formula $\theta_{n+1} = \frac{180}{\pi} (0.8 + \sin \theta_n)$ to find θ correct to 1 decimal place.

$\theta =$ _____

[2 marks]

- (c) Work out the depth of the oil in the tank.
Give your answer to the nearest centimetre.

Answer _____ cm

[2 marks]

END OF QUESTIONS