

Functions and iteration

Foundation and Higher tiers | Total marks: 89

Marks

- M** Method mark, for a correct method. It can be implied by a correct answer.
- A** Accuracy mark, for a correct answer. It needs the method mark before it.
- B** A mark for a correct value or statement, with or without working.
- ft** Follow through: the mark is given for using an earlier wrong answer correctly.
- oe** Or equivalent.
- [a, b]** Any value from a to b inclusive.

Questions

Q	Skill	Spec	Tier	Grade	Calculator	Marks
1	Function machines	A7	Foundation	2	No	3
2	Inputs and outputs	A7	Foundation	3	No	3
3	Function machines with algebra	A7	Foundation	4	No	3
4	Comparing two function machines	A7 A17	Common	5	No	3
5	Function notation	A7	Higher	6	No	3
6	Inverse functions	A7	Higher	7	No	3
7	Functions from a graph	A7	Higher	7	No	3
8	Composite functions	A7	Higher	8	No	4
9	Working backwards from a composite function	A7	Higher	8	No	2
10	Inverse of a function	A7	Higher	9	No	3
11	Composite functions and a turning point	A7 A11	Higher	9	No	5
12	Composite functions with a reciprocal	A7 A18	Higher	9	No	4
13	Iteration formulae	A20	Higher	7	Yes	4
14	Iteration and a change of sign	A20	Higher	8	Yes	7
15	Composite and inverse functions with decimals	A7 A17	Higher	8	Yes	5
16	Iteration in a context	A20	Higher	8	Yes	5

Q	Skill	Spec	Tier	Grade	Calculator	Marks
17	Choosing an iteration formula, then a change of sign	A20	Higher	9	Yes	5
18	Forming an equation, then iteration to a given accuracy	A20 G16	Higher	9	Yes	6
19	Working back from a composite function to its inverse	A7	Higher	9	Yes	6
20	Iteration for an interest rate	A20 R16	Higher	9	Yes	6
21	Iteration with a sector and a triangle	A20 G18 G23	Higher	9	Yes	6
	Total					89

Question 1 A7 · Foundation · Grade 2 · 3 marks

Q	Answer	Mark	Comments
1(a)	16	B1	
1(b)	$22 + 5$ or 27	M1	oe, then $\div 3$
	9	A1	

Question 2 A7 · Foundation · Grade 3 · 3 marks

Q	Answer	Mark	Comments
2	3	B1	output for 10
	18	B1	input for 7
	-3	B1	output for -2

Question 3 A7 · Foundation · Grade 4 · 3 marks

Q	Answer	Mark	Comments
3(a)	$4n + 3$	B1	oe
3(b)	$4n$ is always even	M1	oe, e.g. 4 times any whole number is a multiple of 4, so even
	even + 3 is always odd	A1	a full explanation

Question 4 A7 A17 · Common · Grade 5 · 3 marks

Q	Answer	Mark	Comments
4	$2x + 9$ or $5(x + 3)$	M1	oe, either expression
	$2x + 9 = 5(x + 3)$	M1	oe, or $2x + 9 = 5x + 15$
	-2	A1	

Question 5 A7 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
5(a)	7	B1	
5(b)	$3x^2 = 75$ or $x^2 = 25$	M1	oe
	5 and -5	A1	

Question 6 A7 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
6(a)	$y = \frac{2x-1}{5}$ rearranged to $5y = 2x - 1$	M1	oe, or reverses the steps: $\times 5$ then $+ 1$ then $\div 2$
	$f^{-1}(x) = \frac{5x+1}{2}$	A1	oe
6(b)	8	B1ft	ft their inverse

Question 7 A7 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
7(a)	3.5	B1	accept [3.4, 3.6]
7(b)	2	B1	accept [1.9, 2.1]
7(c)	-0.5	B1	accept [-0.6, -0.4]

Question 8 A7 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
8(a)	13	B1	
8(b)	$(x + 4)^2$	B1	oe
8(c)	$x^2 + 4 = x^2 + 8x + 16$	M1	oe, ft their gf(x)
	-1.5	A1	oe, $-\frac{3}{2}$

Question 9 A7 · Higher · Grade 8 · 2 marks

Q	Answer	Mark	Comments
9	$2g(x) + 1 = 6x - 7$	M1	oe, or $\frac{6x-7-1}{2}$
	$g(x) = 3x - 4$	A1	

Question 10 A7 · Higher · Grade 9 · 3 marks

Q	Answer	Mark	Comments
10	$y(x - 1) = x + 6$	M1	oe, from $y = \frac{x+6}{x-1}$, or with x and y swapped
	$xy - x = y + 6$ then $x(y - 1) = y + 6$	M1	oe, collects the terms in x and factorises
	$f^{-1}(x) = \frac{x+6}{x-1}$, which is $f(x)$	A1	a full method with a conclusion

Question 11 A7 A11 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
11(a)	$fg(x) = (x + k)^2 - 6(x + k)$	M1	oe
	$(x + k - 3)^2 - 9$, or the turning point at $x = 3 - k$	M1	oe, or roots $-k$ and $6 - k$ with the turning point halfway between
	5	A1	
11(b)	$gf(x) = x^2 - 6x + 5$ or $(x - 3)^2 - 4$	M1	oe, ft their k
	(3, -4)	A1	

Question 12 A7 A18 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
12	$fg(x) = \frac{1}{2x-1}$ and $gf(x) = \frac{2}{x} - 1$	M1	both
	$x = (2 - x)(2x - 1)$	M1	oe, clears both fractions
	$2x^2 - 4x + 2 = 0$	M1	oe, three terms = 0
	$2(x - 1)^2 = 0$, so only $x = 1$	A1	or the discriminant is 0, with $x = 1$

Question 13 A20 · Higher · Grade 7 · 4 marks

Q	Answer	Mark	Comments
13(a)	$x_1 = 3.666\ldots$ or $\frac{11}{3}$	M1	
	$x_1 = 3.666\ldots$, $x_2 = 3.7272\ldots$ and $x_3 = 3.7317\ldots$	A1	to at least 3 decimal places
13(b)	$\alpha = 4 - \frac{1}{\alpha}$	M1	oe, $x = 4 - \frac{1}{x}$
	multiplies by α: $\alpha^2 = 4\alpha - 1$, so $\alpha^2 - 4\alpha + 1 = 0$	A1	a full method

Question 14 A20 · Higher · Grade 8 · 7 marks

Q	Answer	Mark	Comments
14(a)	substitutes 2 and 3: -1 and 13	M1	oe
	-1 and 13 with a statement that the sign changes	A1	
14(b)	$x^3 = 5x - 1$ then takes the cube root of both sides	B1	
14(c)	$x_1 = \sqrt[3]{9}$ or 2.0800...	M1	
	$x_1 = 2.0800...$, $x_2 = 2.1104...$ and $x_3 = 2.1218...$	A1	to at least 3 decimal places
14(d)	substitutes 2.115 and 2.125, or 2.125 and 2.135, into $x^3 - 5x + 1$	M1	or continues the iteration to 2.1284...
	No, as the value at 2.125 is negative (-0.029...) and at 2.135 is positive (0.056...), so the solution rounds to 2.13	A1	oe, or No as the iteration settles at 2.128...

Question 15 A7 A17 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
15(a)	$g^{-1}(x) = x - 273.15$	B1	oe
15(b)	$1.8(x - 273.15) + 32$	M1	ft their $g^{-1}(x)$
	$1.8x - 491.67 + 32 = 1.8x - 459.67$	A1	a full method
15(c)	$1.8x - 459.67 = x$	M1	oe, or $0.8x = 459.67$
	574.5875	A1	accept 574.59, 574.6 or 575

Question 16 A20 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
16(a)	$A_1 = 6.8$	M1	
	$A_2 = 9.150...$ and $A_3 = 12.140...$	A1	to at least 2 decimal places
16(b)	$L = 1.4L - 0.008L^2$	M1	oe, or $0.008L^2 = 0.4L$
	50	A1	not 0
16(c)	No, as the area gets closer to 50 m^2, which is less than 60 m^2	B1ft	oe, ft their L

Question 17 A20 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
17(a)	$x_1 = 1.666\ldots$ or $\frac{5}{3}$, $x_2 = 1.1049\ldots$, $x_3 = 0.5581\ldots$	M1	at least two correct
	all three correct with a reason	A1	eg the values are getting smaller and further from 2 and 3, or they are heading for a different solution, near 0.34
17(b)	$x_1 = \sqrt[3]{10} = 2.1544\ldots$ and $x_2 = 2.2190\ldots$	M1	or further values reaching 2.261...
	substitutes 2.255 and 2.265 into $x^3 - 6x + 2$	M1	-0.063... and 0.029...
	2.26 with the change of sign stated	A1	

Question 18 A20 G16 · Higher · Grade 9 · 6 marks

Q	Answer	Mark	Comments
18(a)	$\pi x^2(x+4) = 500$	M1	oe
	$x^2 = \frac{500}{\pi(x+4)}$ then takes the square root of both sides	A1	a full method
18(b)	$x_1 = 4.4603\ldots$ and at least two more values, $x_2 = 4.3372\ldots$, $x_3 = 4.3691\ldots$	M1	values to at least 3 decimal places
	8.4	A1	from a radius of 4.362...
18(c)	substitutes radii 4.35 and 4.45 into $\pi x^2(x+4) - 500$	M1	or into $\pi x^2(x+4)$ to compare with 500: 496.4 and 525.7
	-3.62 and 25.7, a change of sign, so the radius is between 4.35 and 4.45 and the height between 8.35 and 8.45	A1	oe, a full conclusion

Question 19 A7 · Higher · Grade 9 · 6 marks

Q	Answer	Mark	Comments
19(a)	$ff(x) = a(ax + b) + b$	M1	oe, or $a^2x + ab + b$
	$a = 1.35$	A1	from $a^2 = 1.8225$
	$1.35b + b = 5.64$	M1	oe, ft their a
	$b = 2.4$	A1	
19(b)	$1.35x + 2.4 = \frac{x-2.4}{1.35}$	M1	oe, ft their a and b , or $1.35x + 2.4 = x$
	-6.86	A1	accept $[-6.858, -6.857]$ or $-\frac{48}{7}$

Question 20 A20 R16 · Higher · Grade 9 · 6 marks

Q	Answer	Mark	Comments
20(a)	$2500m^3$ or $2500m^2$	M1	the first or second payment with its interest
	$2500m^3 + 2500m^2 + 2500m = 8100$, then divides by 2500	A1	a full method
20(b)	$m^2(m+1) = 3.24 - m$, then divides by $m+1$ and takes the square root	B1	a full method
20(c)	$m_1 = 1.0583\dots$	M1	$\sqrt{\frac{2.24}{2}}$
	continues until the values settle at 1.0389...	M1	eg $m_2 = 1.0295\dots$, $m_3 = 1.0436\dots$
	3.9	A1	

Question 21 A20 G18 G23 · Higher · Grade 9 · 6 marks

Q	Answer	Mark	Comments
21(a)	$\frac{\theta}{360} \times \pi \times 1^2 - \frac{1}{2} \times 1 \times 1 \times \sin \theta = 0.4$	M1	oe, sector minus triangle
	$\pi\theta = 360(0.4 + 0.5 \sin \theta)$ rearranged to $\theta = \frac{180}{\pi}(0.8 + \sin \theta)$	A1	a full method
21(b)	$\theta_1 = 102.261\dots$	M1	and at least one more value, $\theta_2 = 101.825\dots$
	101.9	A1	
21(c)	$\cos(\frac{\theta}{2})$ or $1 \times \cos(50.95\dots)$	M1	the distance from O to AB, 0.629... m, ft their θ
	37	A1	accept 0.37 m