

Circle theorems

Higher tier

Total marks: 52

Marks

- M** Method mark, for a correct method. It can be implied by a correct answer.
- A** Accuracy mark, for a correct answer. It needs the method mark before it.
- B** A mark for a correct value or statement, with or without working.
- ft** Follow through: the mark is given for using an earlier wrong answer correctly.
- oe** Or equivalent.
- [a, b]** Any value from a to b inclusive.

Questions

Q	Skill	Spec	Tier	Grade	Calculator	Marks
1	Angle in a semicircle	G10	Higher	5	No	3
2	Tangent and radius	G10 G20	Higher	5	No	4
3	Angle at the centre	G10	Higher	6	No	3
4	Cyclic quadrilateral with algebra	G10 A17	Higher	6	No	3
5	Angles in the same segment	G10	Higher	6	No	3
6	Chord and perpendicular from the centre	G10 G20	Higher	6	No	3
7	Alternate segment theorem	G10	Higher	7	No	3
8	Spot the error: angle at the centre	G10	Higher	7	No	3
9	Tangent, diameter and the same segment	G10	Higher	7	No	3
10	Show that, with two theorems	G10	Higher	8	No	4
11	Prove a circle theorem	G10	Higher	9	No	4
12	Prove with circle theorems: a bisector	G10	Higher	9	No	4
13	Prove the alternate segment theorem	G10	Higher	9	No	4
14	Tangents and trigonometry	G10 G20	Higher	6	Yes	3
15	Tangents, a kite and a sector	G10 G18 G20	Higher	8	Yes	5
	Total					52

Question 1 G10 · Higher · Grade 5 · 3 marks

Q	Answer	Mark	Comments
1	angle ACB = 90°	B1	may be on the diagram
	angle in a semicircle is 90°	B1	and angles in a triangle add up to 180°
	56	B1	

Question 2 G10 G20 · Higher · Grade 5 · 4 marks

Q	Answer	Mark	Comments
2(a)	90	B1	
	the angle between a tangent and a radius is 90°	B1	
2(b)	$5^2 + 12^2$ or 169	M1	oe
	13	A1	

Question 3 G10 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
3	angle AOB = 128°	B1	may be on the diagram
	angle at the centre is twice the angle at the circumference, and triangle OAB is isosceles as OA = OB (radii)	B1	both reasons
	26	B1	

Question 4 G10 A17 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
4	$2x + 10 + 3x - 5 = 180$	M1	oe
	opposite angles of a cyclic quadrilateral add up to 180°	B1	
	35	A1	

Question 5 G10 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
5	angle BDC = 35° or angle ABD = 47°	B1	may be on the diagram
	angles in the same segment are equal	B1	and angles in a triangle add up to 180°
	98	B1	

Question 6 G10 G20 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
6	AM = 15, the perpendicular from the centre to a chord bisects the chord	B1	the reason is needed
	$17^2 - 15^2$ or 64	M1	oe
	8	A1	

Question 7 G10 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
7	angle ABC = 58°	B1	may be on the diagram
	alternate segment theorem	B1	and angles in a triangle add up to 180°
	73	B1	

Question 8 G10 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
8(a)	O is not on the circle so OABC is not a cyclic quadrilateral	B1	oe
8(b)	$360 - 110$ or 250	M1	the reflex angle AOC
	125	A1	

Question 9 G10 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
9	angle DAC = 26° with the angle between a tangent and a radius is 90°	B1	or angle TAC = 90° . Alternative: angle ABC = 90° (angle in a semicircle is 90°) and angle ABD = 64° (alternate segment theorem)
	angles in the same segment are equal	B1	or, in the alternative, $x = 90 - 64$ with both reasons
	26	B1	

Question 10 G10 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
10	PA = PB, tangents from a point to a circle are equal in length	B1	or triangle APB is isosceles
	angle PAB = $\frac{180 - y}{2}$	B1	oe
	angle ACB = angle PAB, alternate segment theorem	B1	
	$\frac{180 - y}{2} = 90 - \frac{y}{2}$	B1	a complete argument with all reasons

Question 11 G10 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
11	angle BAD = a and angle BOD = $2a$	B1	joins OB and OD, angle at the centre is twice the angle at the circumference
	angle BCD = c and reflex angle BOD = $2c$	B1	same reason
	$2a + 2c = 360$	B1	angles around a point add up to 360°
	$a + c = 180$	B1	a complete proof with reasons

Question 12 G10 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
12	angle SBC = angle BAC, alternate segment theorem	B1	or angle TBA = angle BCA
	angle SBC = angle BCA, alternate angles, so angle BAC = angle BCA	B1	or angle TBA = angle BAC
	angle ADB = angle ACB and angle BDC = angle BAC, angles in the same segment are equal	B1	
	angle ADB = angle BDC so BD bisects angle ADC	B1	a complete proof with all reasons

Question 13 G10 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
13	draws the diameter AD and states angle DAS = 90° , the angle between a tangent and a radius is 90°	B1	
	angle ABD = 90° , angle in a semicircle is 90°	B1	
	angle DAB = $90 - x$ so angle ADB = $180 - 90 - (90 - x) = x$	B1	angles in a triangle add up to 180°
	angle ACB = angle ADB = x, angles in the same segment are equal	B1	a complete proof with all reasons

Question 14 G10 G20 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
14	angle OAP = 90°	B1	may be on the diagram, the angle between a tangent and a radius is 90°
	$\sin APO = \frac{4.5}{11.2}$ or $\sin^{-1}(\frac{4.5}{11.2}) = 23.69\dots$	M1	oe
	47.4	A1	accept 47.4 or [47.37, 47.38]

Question 15 G10 G18 G20 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
15	angle AOB = 132°	B1	angles in quadrilateral OATB, with OAT = OBT = 90°
	TA = $\frac{8}{\tan 24}$ or 17.97...	M1	oe
	$2 \times \frac{1}{2} \times 8 \times$ their TA or 143.7...	M1	oe, the area of the kite OATB
	$\frac{132}{360} \times \pi \times 8^2$ or 73.7...	M1	oe, ft their angle AOB
	70.0	A1	accept [69.9, 70.1]