

Upper and lower bounds

Foundation and Higher tiers | Total marks: 109

Marks

- M** Method mark, for a correct method. It can be implied by a correct answer.
- A** Accuracy mark, for a correct answer. It needs the method mark before it.
- B** A mark for a correct value or statement, with or without working.
- ft** Follow through: the mark is given for using an earlier wrong answer correctly.
- oe** Or equivalent.
- [a, b]** Any value from a to b inclusive.

Questions

Q	Skill	Spec	Tier	Grade	Calculator	Marks
1	Limits of accuracy for a count	N16	Common	4	No	2
2	Limits of accuracy for a measurement	N16	Common	5	No	2
3	Interpreting limits of accuracy: will it fit	N16	Common	5	No	2
4	Upper bound of a perimeter	N16	Higher	6	No	2
5	Bounds of a difference	N16	Higher	7	No	3
6	Lower bound of a density	N16 R11	Higher	7	No	3
7	Lower bound of an area	N16	Higher	6	Yes	2
8	Lower bound of a volume	N16 G16	Higher	6	Yes	3
9	Lower bound of an area: can you be sure	N16 G17	Higher	6	Yes	3
10	Lower bound of a distance round a circle	N16	Higher	6	Yes	3
11	An error interval after dividing	N16 N15	Higher	6	Yes	3
12	Lower bound of a speed	N16 R11	Higher	7	Yes	3
13	Bounds in a practical decision	N16	Higher	7	Yes	3
14	Upper bound of a time: a decision	N16 R11	Higher	7	Yes	4
15	Lower bound with Pythagoras	N16 G20	Higher	7	Yes	3
16	Bounds of an area: how many to be sure	N16 G16	Higher	7	Yes	4

Q	Skill	Spec	Tier	Grade	Calculator	Marks
17	Bounds working back from a perimeter	N16 G16	Higher	7	Yes	4
18	Upper bound from a formula	N16 A5	Higher	8	Yes	3
19	Upper bound of an angle	N16 G20	Higher	8	Yes	3
20	Bounds working backwards from an area	N16 G17	Higher	8	Yes	3
21	Upper bound of an angle: a decision	N16 G20	Higher	8	Yes	4
22	Lower bound working back from a volume	N16 G16	Higher	8	Yes	4
23	Upper bound of a percentage change	N16 R9	Higher	8	Yes	3
24	Bounds with standard form	N16 N9	Higher	8	Yes	4
25	Upper bound of a density: a decision	N16 R11 G17	Higher	8	Yes	4
26	A suitable degree of accuracy	N16 R11	Higher	9	Yes	5
27	Bounds with a difference: a suitable degree of accuracy	N16 R11	Higher	9	Yes	5
28	Bounds in a formula: show that	N16 A5	Higher	9	Yes	5
29	Bounds with a quadratic equation	N16 A18	Higher	9	Yes	4
30	Bounds of a slant height: two steps	N16 G17 G20	Higher	9	Yes	5
31	Upper bound with the cosine rule	N16 G22	Higher	9	Yes	4
32	Greatest percentage error from a rounded measurement	N16 G17 R9	Higher	9	Yes	4
	Total					109

Question 1 N16 · Common · Grade 4 · 2 marks

Q	Answer	Mark	Comments
1(a)	2350	B1	
1(b)	2449	B1	

Question 2 N16 · Common · Grade 5 · 2 marks

Q	Answer	Mark	Comments
2(a)	3.35	B1	
2(b)	a value between 3.44 and 3.45, such as 3.445 or 3.449, also rounds to 3.4	B1	oe, such as the mass can be anything up to 3.45

Question 3 N16 · Common · Grade 5 · 2 marks

Q	Answer	Mark	Comments
3	44.5 and 44.65	M1	the least gap and the upper bound of the box
	No, with 44.5 and 44.65 compared	A1	oe, such as the box could be 44.6 cm and the gap 44.5 cm

Question 4 N16 · Higher · Grade 6 · 2 marks

Q	Answer	Mark	Comments
4	7.35	M1	seen
	44.1	A1	

Question 5 N16 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
5(a)	9.45 – 3.75	M1	
	5.7	A1	
5(b)	she should use the upper bound of q , 3.85	B1	oe, such as $9.35 - 3.85 = 5.5$ is smaller

Question 6 N16 R11 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
6	835 and 1250	B1	both seen
	$835 \div 1250$	M1	oe, the lower bound of the mass over the upper bound of the volume
	0.668	A1	

Question 7 N16 · Higher · Grade 6 · 2 marks

Q	Answer	Mark	Comments
7	12.55×8.35	M1	
	104.7925	A1	accept 104.79 or 104.8

Question 8 N16 G16 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
8(a)	6.35^3	M1	oe
	256.047875	A1	accept 256.0 or better
8(b)	No, with a reason	B1ft	such as the lower bound 256.0 is less than 260; ft their (a)

Question 9 N16 G17 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
9	2.35 or 1.175	B1	the lower bound of the diameter or the radius
	$\pi \times 1.175^2$	M1	oe, or $\pi \times 2.35^2 \div 4$
	4.337... and No	A1	accept [4.33, 4.34]

Question 10 N16 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
10	31.5	B1	seen
	$25 \times 2 \times \pi \times 31.5$	M1	oe, or 4948.0...
	4948	A1	accept 4948.01...

Question 11 N16 N15 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
11(a)	$5.25 \div 8$ or $5.35 \div 8$	M1	oe
	$0.65625 \leq p < 0.66875$	A1	accept $\leq$ at the upper end
11(b)	Yes, with 65.625 cm or 0.65625 m	B1ft	ft their lower bound

Question 12 N16 R11 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
12	42.55 or 1.55	M1	seen
	$42.55 \div 1.55$	M1	
	27.5	A1	accept 27.5 or [27.45, 27.46]

Question 13 N16 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
13	825 and 32.5	M1	both seen
	$825 \div 32.5$ or 25.3...	M1	
	25	A1	

Question 14 N16 R11 · Higher · Grade 7 · 4 marks

Q	Answer	Mark	Comments
14	18.45 and 21.5	B1	both seen
	$18.45 \div 21.5$	M1	oe, or 0.858... hours
	their time $\times 60$	M1	oe, or 52 minutes = 0.866... hours
	Yes, with 51.48... minutes	A1	or 0.858... hours and 0.866... hours compared

Question 15 N16 G20 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
15	95.5 and 57.5	B1	both seen
	$\sqrt{(95.5^2 + 57.5^2)}$	M1	oe
	111.5	A1	accept 111.5 or [111.47, 111.48]

Question 16 N16 G16 · Higher · Grade 7 · 4 marks

Q	Answer	Mark	Comments
16	3.45, 5.85 and 2.65	B1	all three seen
	$\frac{1}{2} \times (3.45 + 5.85) \times 2.65$	M1	oe, or 12.3225
	their greatest area $\div 3.95$	M1	oe, or 3.12...
	4	A1	

Question 17 N16 G16 · Higher · Grade 7 · 4 marks

Q	Answer	Mark	Comments
17	14.35	B1	seen
	$14.35 \div 24$ or 0.5979...	M1	oe, or $1435 \div 24 = 59.79...$ cm, one side of a tile
	their side ²	M1	or $(14.35 \div 4)^2 \div 36$
	3575	A1	accept 3575.04...

Question 18 N16 A5 · Higher · Grade 8 · 3 marks

Q	Answer	Mark	Comments
18	12.5, 2.55 and 0.35	B1	all three seen
	$12.5 \div (2.55 - 0.35)$	M1	oe, ft their bounds if the upper bound of V and the least value of $q - r$ are used
	5.68	A1	accept 5.6818...

Question 19 N16 G20 · Higher · Grade 8 · 3 marks

Q	Answer	Mark	Comments
19	5.25 and 9.75	M1	both seen
	$\sin^{-1}\left(\frac{5.25}{9.75}\right)$	M1	oe
	32.6	A1	accept 32.6 or [32.57, 32.58]

Question 20 N16 G17 · Higher · Grade 8 · 3 marks

Q	Answer	Mark	Comments
20	50.5	M1	seen
	$\sqrt{\left(\frac{50.5}{\pi}\right)}$	M1	oe
	4.01	A1	accept 4.0093...

Question 21 N16 G20 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
21	0.855 and 6.05	B1	both seen
	$\tan x = \frac{0.855}{6.05}$	M1	oe
	8.04...	A1	accept [8.04, 8.05]
	No, with 8.04... compared with 8	A1	dep on M1

Question 22 N16 G16 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
22	1775 and 65.5	B1	both seen
	1775 litres = 1 775 000 cm ³	M1	oe, or 65.5 cm = 0.655 m
	$1\,775\,000 \div (\pi \times 65.5^2)$	M1	oe, consistent units
	131.69	A1	accept 131.69... or 1.3169 m

Question 23 N16 R9 · Higher · Grade 8 · 3 marks

Q	Answer	Mark	Comments
23	3150 and 2350	B1	both seen
	$\frac{3150 - 2350}{2350} \times 100$	M1	oe, or $\frac{3150}{2350} = 1.340\dots$
	34.0	A1	accept 34.0 or [34.04, 34.05]

Question 24 N16 N9 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
24	8.15×10^{13} and 2.995×10^5	B1	both seen
	$(8.15 \times 10^{13}) \div (2.995 \times 10^5)$	M1	oe, or 2.72×10^8 seconds
	their seconds $\div (365 \times 24 \times 60 \times 60)$	M1	oe, or 9 years = $2.838\dots \times 10^8$ seconds
	8.63... years, less than 9	A1	or seconds compared, from correct bounds

Question 25 N16 R11 G17 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
25	397.5 and 4.55 or 2.275	B1	the upper bound of the mass and the lower bound of the diameter or radius
	$\frac{4}{3} \times \pi \times 2.275^3$ or 49.3...	M1	oe, the least volume
	$397.5 \div$ their least volume	M1	oe
	No, with 8.059...	A1	accept [8.05, 8.06]

Question 26 N16 R11 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
26	one of 247.5, 248.5, 61.5, 62.5	B1	
	$248.5 \div 61.5$ or 4.0407...	M1	the upper bound of the time
	$247.5 \div 62.5$ or 3.96	M1	the lower bound of the time
	4.0407... and 3.96	A1	
	4.0 with a reason	A1	such as both bounds round to 4.0 to 2 significant figures

Question 27 N16 R11 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
27(a)	457	B1	642.5 – 185.5
27(b)	457 ÷ 499.5 or 0.9149...	M1	the upper bound of the density, ft their (a)
	(641.5 – 186.5) ÷ 500.5 or 455 ÷ 500.5 or 0.9091...	M1	the lower bound of the density
	0.9149... and 0.9091...	A1	
	0.91 with a reason	A1	such as both bounds round to 0.91 to 2 significant figures

Question 28 N16 A5 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
28(a)	0.8505	B1	the upper bound of L
	37.05 ÷ 20 or 1.8525	B1	the lower bound of T
	$\frac{4\pi^2 \times 0.8505}{1.8525^2}$	M1	oe, upper bound of L with lower bound of T
	9.784... and less than 9.81	A1	
28(b)	the timing error of up to 0.05 seconds is shared between 20 swings	B1	oe, such as it makes T more accurate

Question 29 N16 A18 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
29	40.5	B1	seen
	$x(x + 3) = 40.5$ or $x^2 + 3x - 40.5 = 0$	M1	oe
	$\frac{-3 + \sqrt{(3^2 + 4 \times 40.5)}}{2}$	M1	oe, or completing the square
	5.04	A1	accept 5.0383...

Question 30 N16 G17 G20 · Higher · Grade 9 · 5 marks

Q	Answer	Mark	Comments
30	245 and 5.25	B1	both seen; trying 5.15 as well and choosing the smaller slant height is fine
	$245 = \frac{1}{3} \times \pi \times 5.25^2 \times h$	M1	oe, the volume formula with bounds
	$h = 8.488\dots$	A1	the least height
	$\sqrt{(5.25^2 + 8.488\dots^2)}$	M1	oe, ft their least height with the same radius
	9.98	A1	accept 9.9806...; the same radius must be used for the height and for l . 5.15 throughout gives 10.2, which is not the lower bound

Question 31 N16 G22 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
31	140.5, 95.5 and 72.5	B1	all three seen
	$140.5^2 + 95.5^2 - 2 \times 140.5 \times 95.5 \times \cos 72.5$	M1	oe, or 20790.9...
	$\sqrt{(\text{their value})}$	M1	dep on the first M1
	144.2	A1	accept 144.2 or [144.19, 144.20]

Question 32 N16 G17 R9 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
32	21.5 or 10.75	B1	the least diameter or radius
	volume with 22 and volume with 21.5	M1	oe, 5575.3 and 5203.7, or $\frac{22^3}{21.5^3}$
	$\frac{5575.3 - 5203.7}{5203.7} \times 100$	M1	oe
	7.1	A1	accept 7.1 or [7.14, 7.15]; 6.5 from 22.5 alone scores B0 M1 M1 A0