

Algebraic fractions

Higher tier

Total marks: 63

Marks

- M** Method mark, for a correct method. It can be implied by a correct answer.
- A** Accuracy mark, for a correct answer. It needs the method mark before it.
- B** A mark for a correct value or statement, with or without working.
- ft** Follow through: the mark is given for using an earlier wrong answer correctly.
- oe** Or equivalent.
- [a, b]** Any value from a to b inclusive.

Questions

Q	Skill	Spec	Tier	Grade	Calculator	Marks
1	Simplifying by factorising	A4	Higher	6	No	3
2	Adding fractions with number denominators	A4	Higher	6	No	2
3	Spotting a mistake in cancelling	A4	Higher	6	No	2
4	Simplifying with harder factorising	A4	Higher	7	No	3
5	Multiplying algebraic fractions	A4	Higher	7	No	3
6	Subtracting algebraic fractions	A4	Higher	7	No	3
7	Equations with algebraic fractions	A4 A18	Higher	7	No	4
8	Subtracting when a denominator factorises	A4	Higher	8	No	3
9	Show that, then solve	A4 A18	Higher	8	No	5
10	Identities with algebraic fractions	A4	Higher	8	No	4
11	Forming and solving an equation with fractions	A4 A21 A18	Higher	9	No	7
12	Showing an equation with fractions has no solutions	A4 A18	Higher	9	No	4
13	A fraction equal to a linear expression, by the formula	A4 A18	Higher	8	Yes	4
14	Forming an equation from rates of working	A4 A18 A21	Higher	8	Yes	5
15	Simplifying, then solving with the formula	A4 A18	Higher	8	Yes	5
16	A journey there and back with algebraic fractions	A4 A18 A21	Higher	9	Yes	6
	Total					63

Question 1 A4 · Higher · Grade 6 · 3 marks

Q	Answer	Mark	Comments
1	$x(x + 5)$	B1	
	$(x + 5)(x - 5)$	B1	
	$\frac{x}{x-5}$	B1	

Question 2 A4 · Higher · Grade 6 · 2 marks

Q	Answer	Mark	Comments
2	$\frac{3x}{12} + \frac{2(2x-1)}{12}$	M1	oe, both over a common denominator
	$\frac{7x-2}{12}$	A1	

Question 3 A4 · Higher · Grade 6 · 2 marks

Q	Answer	Mark	Comments
3	$(x + 4)(x - 4)$	M1	or a value that shows Ben is wrong, such as $x = 0$ gives 4, not -4
	No and $x + 4$	A1	or No with a correct counter example

Question 4 A4 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
4	$(2x + 1)(x + 3)$	M1	
	$(2x + 1)(2x - 1)$	M1	
	$\frac{x+3}{2x-1}$	A1	

Question 5 A4 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
5	$(x + 3)(x - 3)$	M1	
	Cancels $x - 3$, or $\frac{2x^2}{4x} = \frac{x}{2}$	M1	either seen
	$\frac{x(x+3)}{2}$	A1	oe, such as $\frac{x^2 + 3x}{2}$

Question 6 A4 · Higher · Grade 7 · 3 marks

Q	Answer	Mark	Comments
6	a common denominator of $(x+2)(x-4)$	M1	oe, such as $x^2 - 2x - 8$
	$3(x-4) - (x+2)$ over the common denominator	M1	oe
	$\frac{2(x-7)}{(x+2)(x-4)}$	A1	oe, such as $\frac{2x-14}{x^2-2x-8}$

Question 7 A4 A18 · Higher · Grade 7 · 4 marks

Q	Answer	Mark	Comments
7	$2(x+3) + 3(x+1) = (x+1)(x+3)$	M1	oe, the fractions cleared
	$x^2 - x - 6 = 0$	M1	oe, a three-term quadratic
	$(x-3)(x+2)$	M1	or a correct use of the formula
	$x = -2$ or $x = 3$	A1	

Question 8 A4 · Higher · Grade 8 · 3 marks

Q	Answer	Mark	Comments
8	$x^2 - 4 = (x+2)(x-2)$	M1	or a common denominator such as $(x^2-4)(x+2)$
	$\frac{x-(x-2)}{(x+2)(x-2)}$	M1	oe
	$\frac{2}{x^2-4}$	A1	oe, such as $\frac{2}{(x+2)(x-2)}$

Question 9 A4 A18 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
9(a)	$6x - 7(x-2)$ or $2x(x-2)$	M1	oe, a common denominator used correctly on one side
	$6x - 7x + 14 = 2x^2 - 4x$	M1	oe, the fractions cleared and brackets expanded
	$2x^2 - 3x - 14 = 0$	A1	with full working and no errors
9(b)	$(2x-7)(x+2)$	M1	or a correct use of the formula
	$x = -2$ or $x = 3.5$	A1	oe, such as $x = \frac{7}{2}$

Question 10 A4 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
10	$(x + a)(x + 3) - x(x - 2)$	M1	the numerator over $(x - 2)(x + 3)$
	$x^2 + 3x + ax + 3a - x^2 + 2x$	M1	oe, expanded correctly
	$3a = 6$ or $5 + a = b$	M1	comparing terms
	$a = 2$ and $b = 7$	A1	

Question 11 A4 A21 A18 · Higher · Grade 9 · 7 marks

Q	Answer	Mark	Comments
11(a)	$\frac{36}{x}$ or $\frac{36}{x+1.5}$	M1	the number of bags bought one year
	$\frac{36}{x} - \frac{36}{x+1.5} = 2$	M1	oe
	$36(x + 1.5) - 36x = 2x(x + 1.5)$	M1	oe, the fractions cleared
	$54 = 2x^2 + 3x$ leading to $2x^2 + 3x - 54 = 0$	A1	with full working and no errors
11(b)	$(2x - 9)(x + 6)$	M1	or a correct use of the formula
	$x = 4.5$ and $x = -6$	A1	oe
	£4.50	A1	rejects the negative value

Question 12 A4 A18 · Higher · Grade 9 · 4 marks

Q	Answer	Mark	Comments
12	$(x + 3)(x - 3)$ and $(x + 3)(x - 2)$	M1	both factorised
	$5(x - 3) = x(x - 2)$	M1	oe, from $\frac{x-3}{x-2} = \frac{x}{5}$
	$x^2 - 7x + 15 = 0$ and $(x - 3.5)^2 + 2.75$ or $(-7)^2 - 4 \times 1 \times 15 = -11$	M1	oe, completing the square or the formula
	$(x - 3.5)^2 + 2.75 \geq 2.75$, so it is never 0; or the square root of a negative number, so no solutions	A1	a full method and a conclusion. Without cancelling, a cubic with the root $x = -3$ must have it rejected because it makes the denominator 0

Question 13 A4 A18 · Higher · Grade 8 · 4 marks

Q	Answer	Mark	Comments
13	$5 = (x + 2)(x - 1)$	M1	oe, the fraction cleared
	$x^2 + x - 7 = 0$	M1	oe, a three-term quadratic
	$\frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times -7}}{2 \times 1}$	M1	oe, or $\frac{-1 \pm \sqrt{29}}{2}$
	$x = -3.19$ or $x = 2.19$	A1	accept $[-3.193, -3.192]$ and $[2.192, 2.193]$

Question 14 A4 A18 A21 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
14(a)	$\frac{1}{x} + \frac{1}{x+10} = \frac{1}{18}$	M1	oe
	$18(x+10) + 18x = x(x+10)$	M1	oe, the fractions cleared
	$x^2 + 10x - 36x - 180 = 0$ leading to $x^2 - 26x - 180 = 0$	A1	with full working and no errors
14(b)	$\frac{26 \pm \sqrt{((-26)^2 - 4 \times 1 \times -180)}}{2 \times 1}$	M1	oe, or $(x-13)^2 = 349$
	31.7	A1	accept [31.68, 31.69]; rejects the negative solution

Question 15 A4 A18 · Higher · Grade 8 · 5 marks

Q	Answer	Mark	Comments
15(a)	$(x+2)(x+3)$ or $(x+2)(x-2)$	M1	
	$\frac{x+3}{x-2}$	A1	
15(b)	$x+3 = x(x-2)$	M1	oe, using their answer to (a)
	$x^2 - 3x - 3 = 0$ and $\frac{3 \pm \sqrt{((-3)^2 - 4 \times 1 \times -3)}}{2 \times 1}$	M1	oe, a three-term quadratic and a correct use of the formula
	$x = -0.79$ or $x = 3.79$	A1	accept [-0.792, -0.791] and [3.791, 3.792]

Question 16 A4 A18 A21 · Higher · Grade 9 · 6 marks

Q	Answer	Mark	Comments
16(a)	$\frac{30}{x}$ or $\frac{30}{x-4}$	M1	the time for one way, in hours
	$\frac{30}{x} + \frac{30}{x-4} = 3.5$	M1	oe
	$30(x-4) + 30x = 3.5x(x-4)$ leading to the given equation	A1	with full working and no errors; 3 hours 30 minutes must be used as 3.5
16(b)	$\frac{148 \pm \sqrt{((-148)^2 - 4 \times 7 \times 240)}}{2 \times 7}$	M1	oe, or both values 1.77 and 19.37
	19.4	A1	accept [19.37, 19.38], with 1.77 rejected because the speed home, $x-4$, would be negative
16(c)	1 hour 57 minutes	B1ft	ft $30 \div (\text{their speed} - 4)$