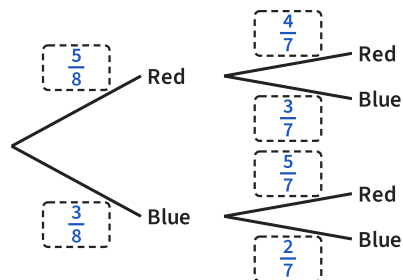


Conditional Probability: Tree Diagrams *extend*

ANSWERS

1

A bag holds 5 red and 3 blue counters. Two are taken without replacement. Given that they are the same colour, find $P(\text{both red})$.

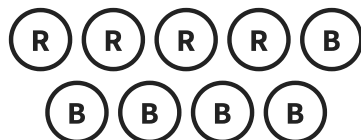


$$\frac{20}{56} \div \left(\frac{20}{56} + \frac{6}{56} \right)$$

Answer $\frac{10}{13}$

2

A bag holds 4 red and 5 blue counters. Three are taken at random, without replacement. Find the probability that more of them are red than blue.



$$3 \text{ orders of 2 red: } 3 \times \frac{4 \times 3 \times 5}{9 \times 8 \times 7} = \frac{180}{504}$$

$$3 \text{ red: } \frac{24}{504}, \text{ total } \frac{204}{504}$$

Answer $\frac{17}{42}$

3

The ratio of red to blue counters in a bag is 3:5. Two counters are taken at random, without replacement. The probability that both are blue is $\frac{5}{14}$. How many counters are in the bag?

$$\frac{5m(m-1)}{8m(8m-1)} = \frac{5}{14}$$

$$70m - 14 = 64m - 8, m = 1$$

Answer 8

4

A bag holds 2 gold and 3 silver coins. Amy and Ben take turns to take a coin at random, without replacement. The first to take a gold coin wins. Amy goes first. Find the probability that Amy wins.

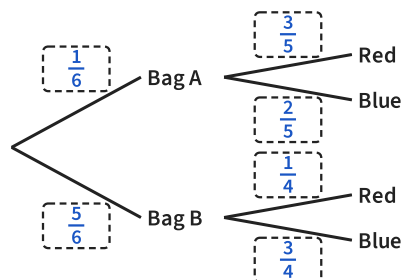
Amy wins on the 1st or 3rd coin

$$\frac{2}{5} + \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = \frac{2}{5} + \frac{1}{5}$$

Answer $\frac{3}{5}$

5

Bag A holds 3 red and 2 blue counters, bag B 1 red and 3 blue. Roll a dice: 6 means bag A, else bag B. Given the counter taken is red, find $P(\text{bag A})$.

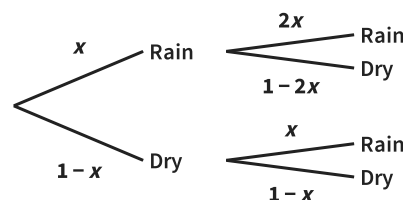


$$\frac{1}{10} \div \left(\frac{1}{10} + \frac{5}{24} \right)$$

Answer $\frac{12}{37}$

6

The tree shows rain on Saturday and Sunday. $P(\text{rain on exactly one day}) = \frac{1}{3}$. Find x .



$$x(1-2x) + (1-x)x = \frac{1}{3}$$

$$2x^2 - 6x + 1 = 0, (3x-1)^2 = 0$$

Answer $x = \frac{1}{3}$

7

A bag holds n red and 6 blue counters. Two are taken at random, without replacement. $P(\text{same colour}) = \frac{1}{2}$.

Show that $n^2 - 13n + 30 = 0$ and find the possible values of n .

$$\frac{n(n-1) + 30}{(n+6)(n+5)} = \frac{1}{2}$$

$$2n^2 - 2n + 60 = n^2 + 11n + 30$$

Answer $n = 3 \text{ or } n = 10$

8

A bag holds r red and b blue counters, where r and b are at least 1. Prove that two taken without replacement are less likely to both be red than two taken with replacement.

$$\text{compare } \frac{r(r-1)}{(r+b)(r+b-1)} \text{ with } \frac{r^2}{(r+b)^2}$$

$$\text{expand: } -r - b < -r, \text{ true as } b \geq 1$$

Proof $\frac{r-1}{r+b-1} < \frac{r}{r+b}$