

Completing the Square *extend*

ANSWERS

1



Find the values of a , b and c for which
 $5 - 8x - 2x^2 \equiv a - b(x + c)^2$

$$-2(x^2 + 4x) + 5$$

$$= -2(x + 2)^2 + 8 + 5$$

$$a = 13, b = 2, c = 2$$

Answer

2



Find the exact coordinates of the
 turning point of $y = (2x - 1)(x + 3)$

$$2x^2 + 5x - 3 = 2(x^2 + \frac{5}{2}x) - 3$$

$$= 2(x + \frac{5}{4})^2 - \frac{25}{8} - 3$$

$$(-\frac{5}{4}, -\frac{49}{8})$$

Answer

3



A translation maps the curve
 $y = x^2 - 10x + 21$ onto the curve
 $y = x^2 + 2x + 5$. Find the vector of
 the translation.

$$\text{turning points } (5, -4) \text{ and } (-1, 4)$$

$$-1 - 5 = -6, 4 - (-4) = 8$$

$$\text{translation by } \begin{pmatrix} -6 \\ 8 \end{pmatrix}$$

Answer

4



Two numbers add up to 10.
 Prove that the sum of their squares is
 at least 50.

$$x^2 + (10 - x)^2 = 2x^2 - 20x + 100$$

$$= 2((x - 5)^2 - 25) + 100$$

$$2(x - 5)^2 + 50 \geq 50$$

Proof

5



P is a point on the line $y = 2x$ and A is
 the point $(5, 0)$. Find the shortest
 possible length of AP .
 Give your answer as a surd.

$$AP^2 = (x - 5)^2 + (2x)^2$$

$$= 5(x - 1)^2 + 20, AP = \sqrt{20}$$

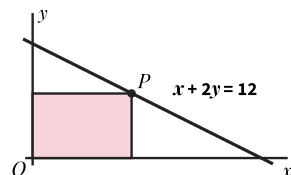
$$2\sqrt{5}, \text{ at } (1, 2)$$

Answer

6



The rectangle has a corner at O and
 sides on the axes. Its corner P is on
 the line $x + 2y = 12$. Find the largest
 possible area of the rectangle.



$$\text{area} = x \times \frac{12 - x}{2} = 6x - \frac{x^2}{2}$$

$$= 18 - \frac{1}{2}(x - 6)^2, \text{ at } P(6, 3)$$

$$18 \text{ square units}$$

Answer

7



The smallest value of
 $x^2 - 2ax + a + 6$ is 0. Find the two
 possible values of a .

$$(x - a)^2 - a^2 + a + 6$$

$$a^2 - a - 6 = 0, (a - 3)(a + 2) = 0$$

$$a = 3 \text{ or } a = -2$$

Answer

8



Show that, whatever the value of k ,
 the turning point of
 $y = x^2 - 4kx + 3k^2 + 2$ lies on the
 curve $y = 2 - \frac{x^2}{4}$

$$(x - 2k)^2 + 2 - k^2$$

$$x = 2k: 2 - \frac{(2k)^2}{4} = 2 - k^2$$

$$(2k, 2 - k^2) \text{ is on it}$$

Reason