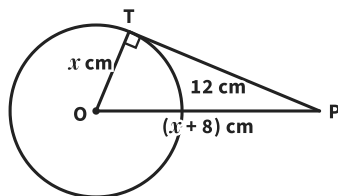


Circle Theorems: Tangents *extend*

ANSWERS

1

PT is a tangent at T. $OT = x$ cm,
 $PT = 12$ cm and $OP = (x + 8)$ cm. Find x .



Not drawn accurately

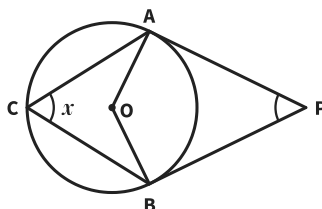
$$OTP = 90^\circ: x^2 + 12^2 = (x + 8)^2$$

$$144 = 16x + 64, \text{ so } x = 5$$

Answer $x = 5$

2

PA and PB are tangents, O is the centre
 and angle $ACB = x$. Show that angle
 $APB = 180 - 2x$.



$$AOB = 2x \text{ (angle at the centre)}$$

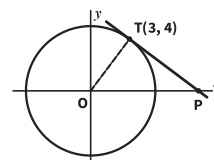
$$OAP = OBP = 90^\circ \text{ (tangent and radius):}$$

$$APB = 360 - 180 - 2x$$

Proof $APB = 180 - 2x$

3

The tangent to the circle $x^2 + y^2 = 25$ at
 $T(3, 4)$ meets the x axis at P.
 Find the exact length of PT.



OT has gradient $\frac{4}{3}$, so the tangent has

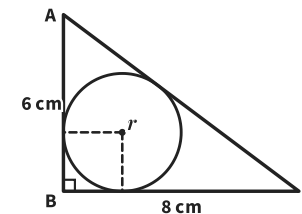
$$\text{gradient } -\frac{3}{4}: y - 4 = -\frac{3}{4}(x - 3)$$

$$y = 0: x = \frac{25}{3}; PT = \sqrt{OP^2 - 5^2} = \frac{20}{3}$$

Answer $PT = \frac{20}{3}$

4

A circle touches all three sides of a right
 angled triangle with shorter sides 6 cm
 and 8 cm. Find the radius of the circle.



Not drawn accurately

hypotenuse = 10 cm; at B the radii and the
 tangents make a square of side r

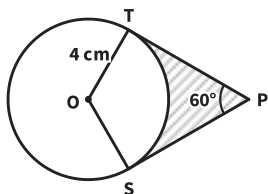
$$\text{equal tangents: } (6 - r) + (8 - r) = 10, \text{ so}$$

$$r = 2$$

Answer $r = 2$ cm

5

PT and PS are tangents, angle $TPS = 60^\circ$
 and the radius is 4 cm. Find the exact
 area shaded.



Not drawn accurately

$$OPT = 30^\circ: PT = \frac{4}{\tan 30^\circ} = 4\sqrt{3};$$

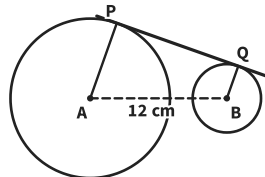
$$\text{kite} = 2 \times \frac{1}{2} \times 4 \times 4\sqrt{3} = 16\sqrt{3}$$

$$TOS = 120^\circ: \text{sector} = \frac{1}{3} \times \pi \times 4^2 = \frac{16\pi}{3}$$

Answer $(16\sqrt{3} - \frac{16\pi}{3}) \text{ cm}^2$

6

Circles with centres A and B have radii
 7 cm and 3 cm, and $AB = 12$ cm.
 A line touches both circles, at P and Q.
 Find the exact length PQ.



Not drawn accurately

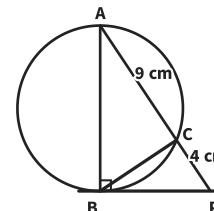
AP and BQ are both at 90° to PQ (tangent
 and radius)

a line from B parallel to PQ makes a right
 angled triangle: $PQ = \sqrt{12^2 - 4^2}$

Answer $PQ = 8\sqrt{2}$ cm

7

AB is a diameter and PB is a tangent
 at B. AP cuts the circle at C, with
 $AC = 9$ cm and $CP = 4$ cm. Find BC.



Not drawn accurately

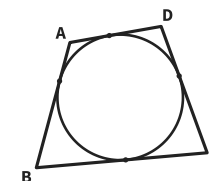
$ACB = 90^\circ$ (semicircle) and $ABP = 90^\circ$
 (tangent and radius)

triangles ACB and BCP are similar:
 $BC^2 = AC \times CP = 36$

Answer $BC = 6$ cm

8

All four sides of the quadrilateral ABCD
 touch the circle.
 Prove that $AB + CD = AD + BC$.



the two tangents from each corner are
 equal: call them a, b, c and d

$$AB + CD = (a + b) + (c + d) = (a + d) +$$

$$(b + c) = AD + BC$$

Proof $AB + CD = AD + BC$