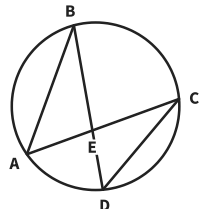


Circle Theorems: Angles in the Same Segment *extend*

ANSWERS

1

AC and BD cross at E. $\angle BAC = (2x + y)^\circ$,
 $\angle BDC = (3x - y)^\circ$, $\angle ABD = (x + y)^\circ$ and
 $\angle ACD = (4y - 10)^\circ$. Find x , y and angle AEB.



Not drawn accurately

$$2x + y = 3x - y, \text{ so } x = 2y;$$

$$x + y = 4y - 10$$

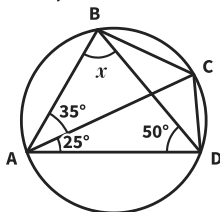
$$3y = 4y - 10; y = 10, x = 20;$$

$$\angle AEB = 180 - 50 - 30$$

Answer $x = 20, y = 10, \angle AEB = 100^\circ$

2

A, B, C and D are on the circle.
 Find angle ABD, marked x .



Not drawn accurately

$$\angle ACB = \angle ADB = 50^\circ \text{ (same segment);}$$

$$\angle BCD = 180 - 60 = 120^\circ$$

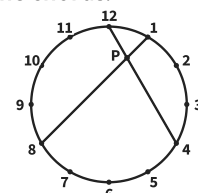
$$\text{(cyclic quadrilateral)}$$

$$x = \angle ACD = 120 - 50 = 70^\circ \text{ (same segment)}$$

Answer $x = 70^\circ$

3

A clock face has a point at each hour on its edge. Chords join 12 to 4 and 1 to 8, crossing at P. Find the acute angle between the chords.



Not drawn accurately

each hour gap makes 30° at the centre, so
 15° at the edge

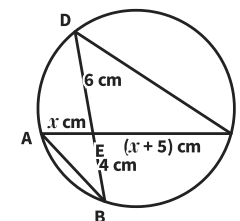
$$\text{at } 12: 3 \times 15 = 45^\circ; \text{ at } 1: 4 \times 15 = 60^\circ;$$

$$180 - 45 - 60 = 75^\circ$$

Answer 75°

4

Chords AC and BD cross at E.
 Use similar triangles to find x .



Not drawn accurately

$\angle BAC = \angle BDC$ and $\angle ABD = \angle ACD$ (same segment), so the triangles are similar

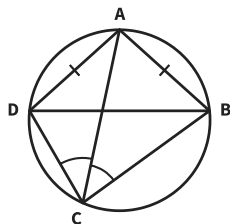
$$\frac{x}{6} = \frac{4}{x+5}; x^2 + 5x - 24 = 0,$$

$$(x+8)(x-3) = 0$$

Answer $x = 3$

5

A, B, C and D are on the circle and $AB = AD$. Prove that AC cuts angle BCD in half.



$$\angle ACB = \angle ADB \text{ (same segment, on AB) and}$$

$$\angle ACD = \angle ABD \text{ (same segment, on AD)}$$

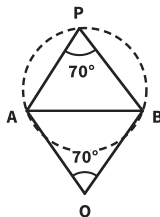
$$\angle ADB = \angle ABD \text{ (triangle ABD is isosceles), so}$$

$$\angle ACB = \angle ACD$$

Proof $\angle ACB = \angle ACD$

6

P and Q are on opposite sides of AB, and $\angle APB = \angle AQB = 70^\circ$. Can A, P, B and Q all lie on one circle? Explain.



Not drawn accurately

APBQ would be a cyclic quadrilateral with $\angle APB$ and $\angle AQB$ opposite

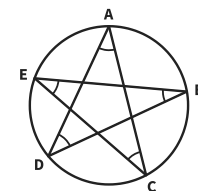
$$\text{opposite angles add to } 180^\circ, \text{ but}$$

$$70 + 70 = 140^\circ$$

Answer No

7

Five points on a circle are joined to make a star. Prove that the five angles at the points of the star add up to 180° .



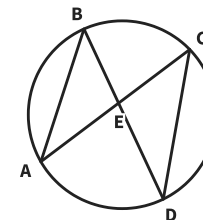
each point's angle stands on one of the five arcs between neighbouring points

each is half the arc's angle at the centre;
 the arcs make 360° , so the sum is
 $\frac{1}{2} \times 360^\circ$

Proof the sum is 180°

8

Chords AC and BD cross at E.
 Prove that $AE \times EC = BE \times ED$.



$\angle BAE = \angle CDE$ and $\angle ABE = \angle DCE$ (angles in the same segment), so triangles ABE and DCE are similar

$$\frac{AE}{DE} = \frac{BE}{CE}, \text{ so } AE \times EC = BE \times ED$$

Proof $AE \times EC = BE \times ED$