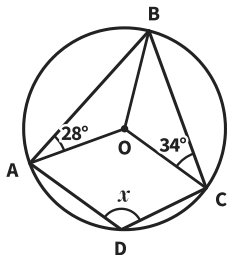


Circle Theorems: Cyclic Quadrilaterals *extend*

ANSWERS

1

O is the centre of the circle.
Find angle ADC, marked x .



Not drawn accurately

$OA = OB = OC$: $OBA = 28^\circ$ and $OBC = 34^\circ$, so
 $ABC = 62^\circ$

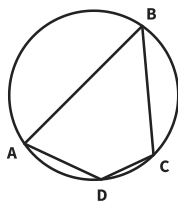
$x = 180 - 62$ (opposite angles of a
cyclic quadrilateral)

$$x = 118^\circ$$

Answer

2

ABCD is a cyclic quadrilateral with angles
 $A = (2x + y)^\circ$, $B = (4x - y)^\circ$, $C = (x + 3y)^\circ$
and $D = (3x + 2y + 10)^\circ$. Find x and y .



Not drawn accurately

$$A + C = 180: 3x + 4y = 180; B + D = 180:$$

$$7x + y = 170$$

$$y = 170 - 7x: 3x + 680 - 28x = 180,$$

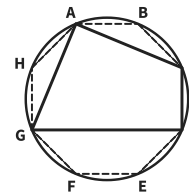
$$x = 20$$

$$x = 20, y = 30$$

Answer

3

ABCDEFGH is a regular octagon in
a circle. Find angles ACD and AGD of the
quadrilateral ACDG.



Not drawn accurately

each side makes 45° at the centre; ACD
faces the arc DEFGHA: $5 \times 45 = 225^\circ$, half
is 112.5°

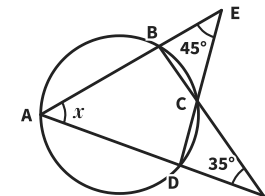
$$AGD = 180 - 112.5 \text{ (opposite angles)}$$

$$ACD = 112.5^\circ, AGD = 67.5^\circ$$

Answer

4

ABCD is cyclic. AB and DC extended meet
at E; AD and BC extended meet at F.
Find angle DAB, marked x .



Not drawn accurately

triangle ADE: $x + ADC = 135$; triangle
ABF: $x + ABC = 145$

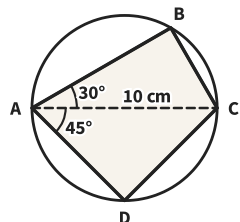
$$\text{add: } 2x + 180 = 280 \text{ (ADC + ABC = 180),}$$

$$x = 50^\circ$$

Answer

5

ABCD is a cyclic quadrilateral. AC = 10 cm
is a diameter. Angle BAC = 30° and angle
CAD = 45° . Find the exact area of ABCD.



Not drawn accurately

$ABC = ADC = 90^\circ$ (semicircle): $AB = 5\sqrt{3}$,
 $BC = 5$, $AD = DC = 5\sqrt{2}$

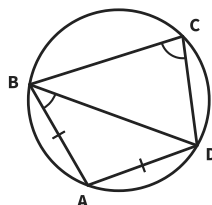
$$\frac{1}{2} \times 5\sqrt{3} \times 5 + \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2}$$

$$\left(25 + \frac{25\sqrt{3}}{2}\right) \text{ cm}^2$$

Answer

6

ABCD is a cyclic quadrilateral with
 $AB = AD$. Show that angle ABD is half of
angle BCD.



$BAD = 180 - BCD$ (opposite angles of a
cyclic quadrilateral)

$$ABD = (180 - BAD) \div 2 = BCD \div 2$$

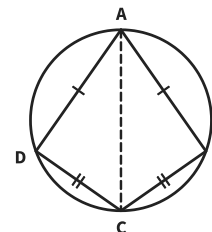
(triangle ABD is isosceles)

$$ABD = \frac{1}{2} BCD$$

Proof

7

ABCD is a kite with $AB = AD$ and $CB = CD$.
Its corners are on a circle. Prove that AC
is a diameter.



triangles ABC and ADC are congruent
(SSS), so angle ABC = angle ADC

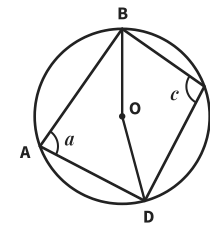
they add to 180° (opposite angles), so each
is 90° : AC is a diameter

$$\text{angle } ABC = 90^\circ$$

Proof

8

ABCD is a cyclic quadrilateral and O is
the centre. Prove that angle BAD + angle
BCD = 180° .



angle BOD = $2a$ and the reflex angle
BOD = $2c$ (angle at the centre)

$$2a + 2c = 360^\circ \text{ (angles at a point), so}$$

$$a + c = 180^\circ$$

$$a + c = 180^\circ$$

Proof