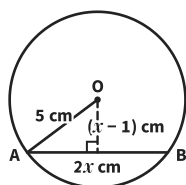


# Circle Theorems: Chords *extend*

ANSWERS 

1

The radius is 5 cm. The chord AB is  $2x$  cm long and  $(x - 1)$  cm from the centre O. Find the length of AB.



Not drawn accurately

$$x^2 + (x - 1)^2 = 5^2 \text{ (half of AB is } x\text{)}$$

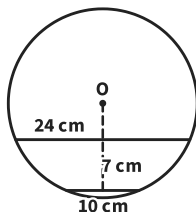
$$2x^2 - 2x - 24 = 0, (x - 4)(x + 3) = 0, \\ x = 4$$

$$AB = 8 \text{ cm}$$

Answer

2

Parallel chords 24 cm and 10 cm long are 7 cm apart, on the same side of the centre. Find the radius.



Not drawn accurately

$$r^2 = 12^2 + d^2 = 5^2 + (d + 7)^2$$

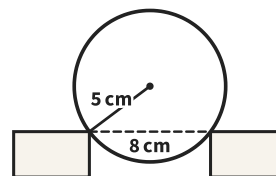
$$144 = 14d + 74: d = 5, r = 13$$

$$13 \text{ cm}$$

Answer

3

A ball of radius 5 cm rests in a round hole of diameter 8 cm in a flat board. How far below the top of the board does the ball reach?



Not drawn accurately

the rim is a chord 8 cm long; half of it is 4 cm

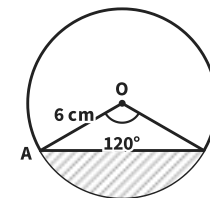
$$\text{centre } \sqrt{5^2 - 4^2} = 3 \text{ cm above it; } 5 - 3 = 2$$

$$2 \text{ cm}$$

Answer

4

Angle AOB =  $120^\circ$  and the radius is 6 cm. Find the exact area of the shaded segment.



Not drawn accurately

$$\text{sector} = \frac{1}{3} \times \pi \times 6^2 = 12\pi$$

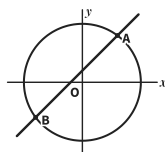
$$\text{triangle} = \frac{1}{2} \times 6 \times 6 \times \sin 120^\circ = 9\sqrt{3}$$

$$(12\pi - 9\sqrt{3}) \text{ cm}^2$$

Answer

5

The line  $y = x + 1$  cuts the circle  $x^2 + y^2 = 25$  at A and B. Find the exact length of AB, then the exact distance from O to AB.



$$x^2 + (x + 1)^2 = 25: x = 3 \text{ or } -4, A(3, 4), \\ B(-4, -3)$$

$$AB = \sqrt{7^2 + 7^2} = 7\sqrt{2};$$

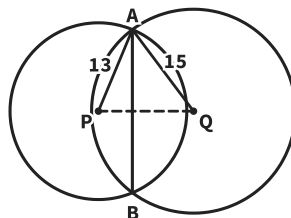
$$\text{distance} = \sqrt{25 - 24.5} = \frac{\sqrt{2}}{2}$$

$$AB = 7\sqrt{2}, \text{ distance } \frac{\sqrt{2}}{2}$$

Answer

6

Circles with centres P and Q and radii 13 cm and 15 cm cross at A and B. AB = 24 cm. Find the distance PQ.



Not drawn accurately

PQ cuts AB in half at  $90^\circ$ , so half of AB is 12 cm

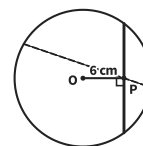
$$\sqrt{13^2 - 12^2} + \sqrt{15^2 - 12^2} = 5 + 9$$

$$PQ = 14 \text{ cm}$$

Answer

7

P is 6 cm from the centre O of a circle of radius 10 cm. Find the length of the shortest chord through P, and explain why.



Not drawn accurately

the further a chord is from O, the shorter it is; through P, the furthest is OP, at  $90^\circ$  to OP

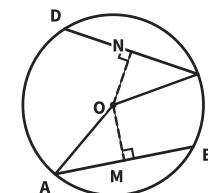
$$2 \times \sqrt{10^2 - 6^2} = 2 \times 8$$

$$16 \text{ cm}$$

Answer

8

AB and CD are chords of equal length, with midpoints M and N. O is the centre. Prove that OM = ON.



$OM \perp AB$  and  $ON \perp CD$ ;  $AM = CN$  (halves of equal chords),  $OA = OC$  (radii)

$$OM^2 = OA^2 - AM^2 = OC^2 - CN^2 = ON^2 \\ \text{(Pythagoras)}$$

$$OM = ON$$

Proof