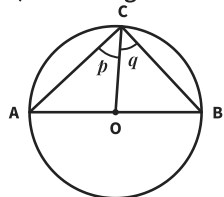


Circle Theorems: Angle in a Semicircle *extend*

ANSWERS

1

AB is a diameter and O is the centre.
Angle $p = (2x + 5)^\circ$ and angle
 $q = (3x - 10)^\circ$. Find angle AOC.



Not drawn accurately

$$ACB = 90^\circ: (2x + 5) + (3x - 10) = 90, \text{ so } x = 19$$

$$OCA = 43^\circ = OAC \text{ (OA = OC),}$$

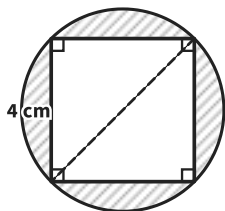
$$AOC = 180 - 2 \times 43$$

$$AOC = 94^\circ$$

Answer

2

A square of side 4 cm has its corners on a circle. Find the exact area shaded.



Not drawn accurately

each corner is 90° , so a diagonal is a diameter: $4\sqrt{2}$ cm (angle in a semicircle)

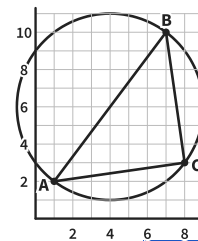
$$\text{radius } 2\sqrt{2}: \pi(2\sqrt{2})^2 - 4^2 = 8\pi - 16$$

$$(8\pi - 16) \text{ cm}^2$$

Answer

3

AB is a diameter, A(1, 2), B(7, 10).
Show that C(8, 3) is on the circle and
find the area of ABC.



centre (4, 6), radius $\sqrt{3^2 + 4^2} = 5$; C is
 $\sqrt{4^2 + 3^2} = 5$ from the centre

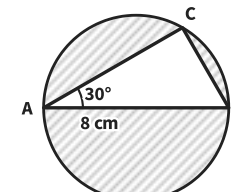
$$ACB = 90^\circ: \text{area} = \frac{1}{2} \times \sqrt{50} \times \sqrt{50} = 25$$

$$\text{area} = 25$$

Answer

4

AB is a diameter of length 8 cm and
angle $CAB = 30^\circ$. Find the exact
area shaded.



Not drawn accurately

$$ACB = 90^\circ: BC = 8 \sin 30^\circ = 4, AC = 8 \cos 30^\circ = 4\sqrt{3}$$

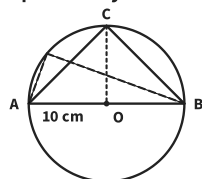
$$16\pi - \frac{1}{2} \times 4 \times 4\sqrt{3} = 16\pi - 8\sqrt{3}$$

$$(16\pi - 8\sqrt{3}) \text{ cm}^2$$

Answer

5

AB is a diameter of length 10 cm and C
moves round the circle.
Find the greatest area triangle ABC
can have. Explain why.



Not drawn accurately

area = $\frac{1}{2} \times AB \times \text{height}$; the height is
greatest with C at the top, the radius 5 cm

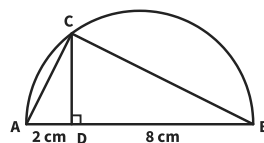
$$\frac{1}{2} \times 10 \times 5 = 25 \text{ (C is then above O and angle ACB is still } 90^\circ)$$

$$25 \text{ cm}^2$$

Answer

6

AB is a diameter (only half the circle is
drawn) and CD is perpendicular to AB.
Show that triangles ADC and CDB are
similar, then find CD.



Not drawn accurately

$ACB = 90^\circ$, so angle
 $ACD = 90 - CAD = \text{angle } CBD$; both have
 90° at D

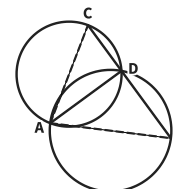
$$\frac{CD}{AD} = \frac{DB}{CD}, \text{ so } CD^2 = 2 \times 8 = 16$$

$$CD = 4 \text{ cm}$$

Answer

7

Two circles have diameters AB and
AC (dashed). They meet at A and D.
Prove that B, D and C lie on a
straight line.



$ADB = 90^\circ$ (angle in a semicircle, in the
circle on AB)

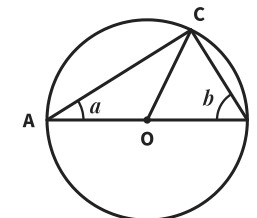
$ADC = 90^\circ$ (the circle on AC):
 $90 + 90 = 180$, so BDC is straight

BDC is a straight line

Proof

8

AB is a diameter and O is the centre.
Prove that angle $ACB = 90^\circ$.



$OA = OC$ and $OB = OC$, so $OCA = a$ and
 $OCB = b$

triangle ABC: $a + b + (a + b) = 180$, so
 $ACB = a + b = 90^\circ$

$$ACB = 90^\circ$$

Proof