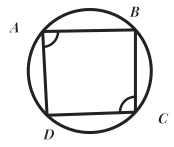
Circle theorems and algebra

ANSWERS

1

$ABCD$ is a cyclic quadrilateral. Angle $DAB = (3x + 10)°$ and angle $BCD = (2x + 20)°$. Find x.



Theorem: Cyclic quadrilateral

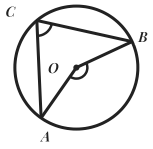
$3x + 10 + 2x + 20 = 180$

$x = 30$

Answer $x = 30$

2

O is the centre. Angle $AOB = (3x + 45)°$ and angle $ACB = (x + 40)°$. Find x.



Theorem: Angle at the centre

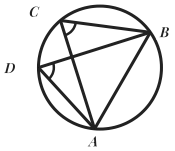
$3x + 45 = 2(x + 40)$

$x = 35$

Answer $x = 35$

3

Angle $ACB = (2x + 15)°$ and angle $ADB = (5x - 30)°$. Find x.



Theorem: Same segment

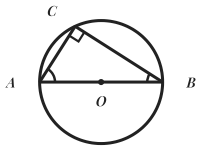
$2x + 15 = 5x - 30$

$x = 15$

Answer $x = 15$

4

AB is a diameter. Angle $CAB = x°$ and angle $CBA = (2x + 15)°$. Find x.

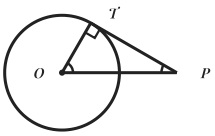


$x = 25$

Answer $x = 25$

5

PT is a tangent at T and O is the centre. Angle $TOP = 3x°$ and angle $OPT = (x + 10)°$. Find x.

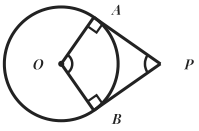


$OTP = 90°$: $3x + x + 10 + 90 = 180$

Answer $x = 20$

6

PA and PB are tangents and O is the centre. Angle $APB = (x + 40)°$ and angle $AOB = 3x°$. Find x.

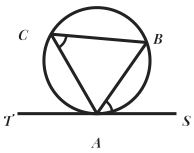


the angles in $OAPB$ add to 360°, with two right angles: $(x + 40) + 3x = 180$

Answer $x = 35$

7

TS is a tangent at A. Angle $SAB = (2x + 8)°$ and angle $ACB = (3x - 12)°$. Find x.

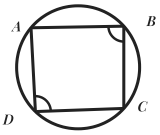


alternate segment: $2x + 8 = 3x - 12$

Answer $x = 20$

8

$ABCD$ is a cyclic quadrilateral. Angle $ABC = (2x + 16)°$ and angle $CDA = (3x - 6)°$. Find the size of angle ABC.



$5x + 10 = 180$, $x = 34$, then $2 \times 34 + 16$

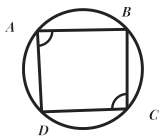
Answer 84°

Circle theorems and algebra *further practice*

ANSWERS

9

$ABCD$ is a cyclic quadrilateral. Angle $DAB = (5x - 4)°$ and angle $BCD = (3x + 24)°$. Find x.

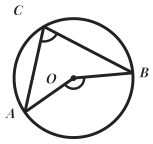


$8x + 20 = 180$

Answer $x = 20$

10

O is the centre. Angle $AOB = (7x + 10)°$ and angle $ACB = (3x + 15)°$. Find x, then angle AOB.



$7x + 10 = 2(3x + 15)$

Answer $x = 20$, 150°

11

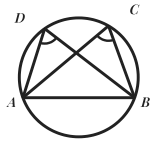
PQ is a diameter of a circle and R is a point on the circle. Angle $QPR = (3x - 5)°$ and angle $PQR = (2x + 20)°$. Find x.

angle $PRQ = 90°$, so $5x + 15 = 90$

Answer $x = 15$

12

Angle $ACB = (7x - 22)°$ and angle $ADB = (4x + 11)°$. Find x, then the size of each angle.



same segment: $7x - 22 = 4x + 11$

Answer $x = 11$, 55°

13

$ABCD$ is a cyclic quadrilateral and CD is extended to E. Angle $ADE = (4x + 5)°$ and angle $ABC = (6x - 25)°$. Find x.

$ADC = 180° - ADE$, and $ADC + ABC = 180°$, so $ADE = ABC$

Answer $x = 15$

14

TS is a tangent at A and O is the centre. The angle between the tangent and the chord AB is $(2x + 10)°$. Angle AOB is $5x°$. Find x.

the angle at the circumference in the other segment is half of $5x$, and equals $2x + 10$

Answer $x = 20$

15

$ABCD$ is a cyclic quadrilateral. Angle $A = 3x$, angle $B = 2y + 10$, angle $C = x + 2y$ and angle $D = x + 50$. Find x and y.

$4x + 2y = 180$ and $x + 2y = 120$

Answer $x = 20$, $y = 50$

16

The angle at the centre is $4x$ and the angle at the circumference is $x + 25$, from the same arc. Jo writes $2 \times 4x = x + 25$. Explain her mistake and find x.

the angle at the centre is twice the other, so $4x = 2(x + 25)$

Answer $x = 25$