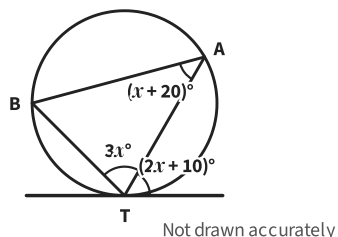


Circle Theorems: Alternate Segment *extend*

ANSWERS

1

The line is a tangent to the circle at T. Find the value of x and the size of angle TBA.



$$TBA = 2x + 10 \text{ (alternate segment)}$$

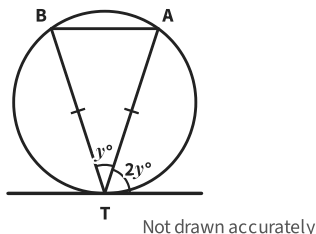
$$(2x + 10) + 3x + (x + 20) = 180, 6x = 150$$

$$x = 25, TBA = 60^\circ$$

Answer

2

TA = TB and the line is a tangent at T. The angle between the tangent and TA is twice angle ATB. Find angle ATB.



$$TBA = 2y \text{ (alternate segment), } TAB = 2y \text{ (TA = TB)}$$

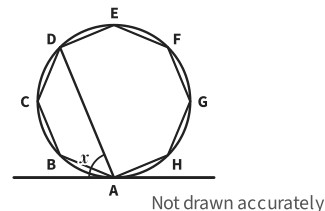
$$2y + 2y + y = 180, \text{ so } y = 36$$

$$ATB = 36^\circ$$

Answer

3

ABCDEFGH is a regular octagon in a circle, with a tangent at A. Find angle x between the tangent and AD.



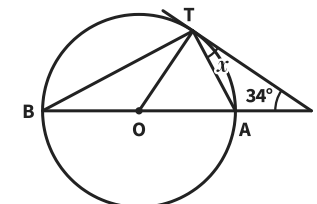
$$\begin{aligned} \text{each side makes } 360 \div 8 = 45^\circ \text{ at the centre, so angle } AOD &= 135^\circ \\ \text{angle } AFD &= 135 \div 2 = 67.5^\circ \text{ (angle at the centre), and } x = \text{angle } AFD \\ &\text{(alternate segment)} \end{aligned}$$

$$x = 67.5^\circ$$

Answer

4

PT is a tangent at T and PAB passes through the centre O. Find angle PTA, marked x .



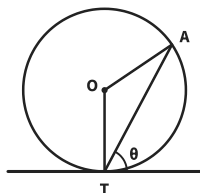
$$\begin{aligned} \text{angle } OTP &= 90^\circ \text{ (tangent and radius), so angle } TOP = 56^\circ \\ \text{angle } TBA &= 56 \div 2 = 28^\circ \text{ (angle at the centre), } x = 28^\circ \text{ (alternate segment)} \end{aligned}$$

$$x = 28^\circ$$

Answer

5

The line is a tangent at T and O is the centre. The angle between the tangent and the chord TA is θ . Show that angle TOA = 2θ .



$$\text{angle } OTA = 90 - \theta \text{ (tangent and radius)}$$

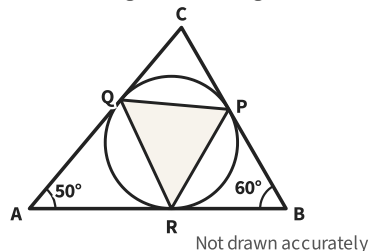
$$OT = OA: \text{ angle } TOA = 180 - 2(90 - \theta) = 2\theta$$

$$\text{angle } TOA = 2\theta$$

Proof

6

The circle touches all three sides of triangle ABC, at P, Q and R. Find the three angles of triangle PQR.



$$\begin{aligned} AR = AQ \text{ (tangents from A), so angle } \\ ARQ = (180 - 50) \div 2 = 65^\circ \end{aligned}$$

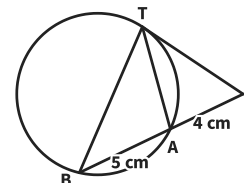
$$QPR = \text{angle } ARQ \text{ (alternate segment); the same at B and C}$$

$$QPR = 65^\circ, PQR = 60^\circ, PRQ = 55^\circ$$

Answer

7

PT is a tangent at T. The line PAB cuts the circle at A and B. Show that triangles PTA and PBT are similar, then find PT.



$$\begin{aligned} \text{angle } PTA &= \text{angle } PBT \text{ (alternate segment),} \\ \text{angle } P &\text{ is shared} \end{aligned}$$

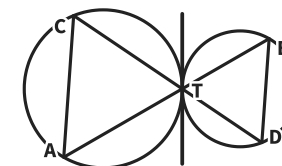
$$\frac{PT}{PA} = \frac{PB}{PT}, \text{ so } PT^2 = 4 \times 9 = 36$$

$$PT = 6 \text{ cm}$$

Answer

8

Two circles touch at T, with a common tangent. ATB and CTD are straight. Prove that AC is parallel to BD.



$$\begin{aligned} ACT = (TA, \text{ tangent}) = (TB, \text{ tangent}) = BDT: \\ \text{alternate segment, vertically opposite} \end{aligned}$$

$$ACT \text{ and } BDT \text{ are alternate angles, so } AC \parallel BD$$

$$AC \parallel BD$$

Proof