Algebraic proof

ANSWERS

1

Write each one using n.

An even number

$2n$

An odd number

$2n + 1$

Three consecutive integers

n , $n + 1$, $n + 2$

2

Write each one using n.

Two consecutive even numbers

$2n$, $2n + 2$

Two consecutive odd numbers

$2n + 1$, $2n + 3$

An even number and the next odd number

$2n$, $2n + 1$

3

Prove that the sum of three consecutive integers is a multiple of 3.

$n + n + 1 + n + 2$

$= 3n + 3$

$= 3(n + 1)$

So it is a multiple of 3

4

Prove that the sum of two consecutive odd numbers is even.

$2n + 1 + 2n + 3$

$= 4n + 4$

$= 2(2n + 2)$

So it is even

5

Prove that the sum of five consecutive integers is a multiple of 5.

$n + n + 1 + n + 2 + n + 3 + n + 4$

$= 5n + 10$

$= 5(n + 2)$

So it is a multiple of 5

6

Prove that $(n + 6)^2 - n^2$ is a multiple of 12.

$n^2 + 12n + 36 - n^2$

$= 12n + 36$

$= 12(n + 3)$

So it is a multiple of 12

7

Prove that $(n + 5)^2 - (n + 3)^2$ is a multiple of 4.

$n^2 + 10n + 25 - (n^2 + 6n + 9)$

$= 4n + 16$

$= 4(n + 4)$

So it is a multiple of 4

8

Kim tries to prove that $(n + 3)^2 - n^2$ is a multiple of 3.
She writes $n^2 + 9 - n^2 = 9 = 3 \times 3$.
Explain her mistake. Then write a correct proof.

$(n + 3)^2$ is not $n^2 + 9$

$n^2 + 6n + 9 - n^2$

$= 6n + 9$

$= 3(2n + 3)$

So it is a multiple of 3

Algebraic proof *further practice*

ANSWERS

9

Prove that the sum of seven consecutive integers is a multiple of 7.

$n + n + 1 + \ldots + n + 6$

$= 7n + 21$

$= 7(n + 3)$

So it is a multiple of 7

10

Jo writes two consecutive odd numbers as $2n + 1$ and $2n + 2$.
Explain her mistake.

$2n + 2 = 2(n + 1)$ is even

the next odd number is $2n + 3$

Answer add 2, not 1

11

Prove that $(n + 5)^2 - (n + 1)^2$ is a multiple of 8.

$n^2 + 10n + 25 - (n^2 + 2n + 1)$

$= 8n + 24$

$= 8(n + 3)$

So it is a multiple of 8

12

Prove that the square of an even number is a multiple of 4.

an even number: $2n$

$(2n)^2 = 4n^2$

So it is 4 × a whole number

13

Kai picks three consecutive integers.
Prove that their sum is 3 times the middle one.

$n + n + 1 + n + 2 = 3n + 3$

$= 3(n + 1)$

$n + 1$ is the middle one

Answer 3 × the middle one

14

Prove that the sum of six consecutive integers is odd.

$n + n + 1 + \ldots + n + 5$

$= 6n + 15$

$= 2(3n + 7) + 1$

So it is 2 × a whole number + 1

15

Pick three consecutive integers.
Square the middle one.
Multiply the other two. Prove that the difference is always 1.

$n, n + 1, n + 2$

$(n + 1)^2 - n(n + 2)$

$= n^2 + 2n + 1 - n^2 - 2n = 1$

Answer 1, for every n

16

Mia works out 2 + 4 = 6, 6 + 8 = 14 and 10 + 12 = 22. She says this proves that the sum of two consecutive even numbers is never a multiple of 4.
Explain why this is not a proof.
Then prove it.

three examples, not every number

$2n + 2n + 2 = 4n + 2$

$4n + 2$: 2 more than a multiple of 4

Answer never a multiple of 4