

1



Prove that the difference between the squares of two consecutive whole numbers is equal to the sum of the two numbers.

$$(n+1)^2 - n^2 = 2n + 1$$

$$n + (n+1) = 2n + 1$$

Reason *both are $2n + 1$*

2



Prove that any three digit number, take away the sum of its digits, gives a multiple of 9.

$$100a + 10b + c - (a + b + c)$$

$$= 99a + 9b = 9(11a + b)$$

Proof *$9(11a + b)$*

3



Prove that the sum of the squares of two consecutive odd numbers is always 2 more than a multiple of 8.

$$(2n+1)^2 + (2n+3)^2 = 8n^2 + 16n + 10$$

$$= 8(n^2 + 2n + 1) + 2$$

Proof *$8(n+1)^2 + 2$*

4



Prove that $(3n+1)^2 - (n-1)^2$ is a multiple of 16 for every whole number n .

$$9n^2 + 6n + 1 - (n^2 - 2n + 1)$$

$$= 8n^2 + 8n = 8n(n+1)$$

$$n(n+1) \text{ is even, so } 8 \times \text{even}$$

Reason *$8 \times \text{an even number}$*

5



Prove that the square of any odd number is 1 more than a multiple of 8.

$$(2n+1)^2 = 4n^2 + 4n + 1$$

$$= 4n(n+1) + 1$$

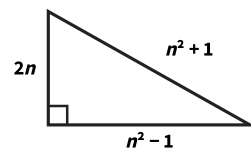
$$n(n+1) \text{ is even, so } 4 \times \text{even}$$

Reason *$4n(n+1) + 1$*

6



A triangle has sides $n^2 - 1$, $2n$ and $n^2 + 1$, where $n > 1$. Prove that it is right angled.



$$(n^2 - 1)^2 + (2n)^2 = n^4 + 2n^2 + 1$$

$$= (n^2 + 1)^2$$

Reason *Pythagoras holds*

7



Prove that $x^2 - 6x + 11$ is positive for every value of x .

$$x^2 - 6x + 11 = (x-3)^2 + 2$$

$$(x-3)^2 \geq 0, \text{ so it is at least } 2$$

Reason *$(x-3)^2 + 2 \geq 2$*

8



Prove that the product of four consecutive whole numbers, plus 1, is always a square number.

$$n(n+3) = n^2 + 3n$$

$$(n+1)(n+2) = n^2 + 3n + 2$$

$$k = n^2 + 3n: k(k+2) + 1 = (k+1)^2$$

Proof *$(n^2 + 3n + 1)^2$*