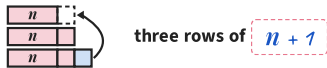


1

Prove that the sum of three consecutive integers is a multiple of 3.

move one square from the longest row to the shortest



three rows of $n + 1$

write the numbers using n

$$n + n + 1 + n + 2$$

collect like terms

$$= 3n + 3$$

factorise to show it

$$= 3(n + 1)$$

So it is a multiple of 3

2

Prove that the sum of two consecutive odd numbers is a multiple of 4.

move one square from the longest row to the shortest



two rows of $2n + 2$

write the numbers using n

$$2n + 1 + 2n + 3$$

collect like terms

$$= 4n + 4$$

factorise to show it

$$= 4(n + 1)$$

So it is a multiple of 4

3

Prove that the sum of four consecutive integers is even.

write the numbers using n

$$n + n + 1 + n + 2 + n + 3$$

collect like terms

$$= 4n + 6$$

factorise to show it

$$= 2(2n + 3)$$

So it is $2 \times$ a whole number

4

Prove that the sum of three consecutive even numbers is a multiple of 6.

write the numbers using n

$$2n + 2n + 2 + 2n + 4$$

collect like terms

$$= 6n + 6$$

factorise to show it

$$= 6(n + 1)$$

So it is a multiple of 6

5

Prove that $(n + 3)^2 - n^2$ is a multiple of 3 for every whole number n .

expand the brackets

$$n^2 + 6n + 9 - n^2$$

collect like terms

$$= 6n + 9$$

factorise to show it

$$= 3(2n + 3)$$

So it is a multiple of 3

6

Prove that $(n + 1)^2 - (n - 1)^2$ is a multiple of 4 for every whole number n .

$$n^2 + 2n + 1 - n^2 + 2n - 1$$

$$= 4n$$

$$= 4 \times n$$

So it is a multiple of 4

7

Prove that the square of an odd number is odd.

$$(2n + 1)^2$$

$$= 4n^2 + 4n + 1$$

$$= 2(2n^2 + 2n) + 1$$

So it is 1 more than an even number

8

Prove that the product of two consecutive odd numbers is odd.

$$(2n + 1)(2n + 3)$$

$$= 4n^2 + 8n + 3$$

$$= 2(2n^2 + 4n + 1) + 1$$

So it is 1 more than an even number

Algebraic Proof further practice

9

Prove that the product of two consecutive even numbers is a multiple of 4.

$$2n(2n + 2)$$

$$= 4n^2 + 4n = 4(n^2 + n)$$

Answer $4(n^2 + n)$, a multiple of 4

10

Prove that the difference between the squares of two consecutive odd numbers is a multiple of 8.

$$(2n + 1)^2 - (2n - 1)^2$$

$$= (4n^2 + 4n + 1) - (4n^2 - 4n + 1)$$

$$= 8n$$

Answer $8n$, a multiple of 8

11

Mo says that $n^2 - n + 11$ is prime for every whole number n . Show that Mo is wrong.

$$n = 11: 121 - 11 + 11 = 121$$

Answer $n = 11$ gives $121 = 11 \times 11$

12

A square has sides of $(n + 1)$ cm. A rectangle is $(n + 3)$ cm by $(n - 1)$ cm. Prove that the area of the square is always 4 cm^2 more than the area of the rectangle.

$$(n + 1)^2 = n^2 + 2n + 1$$

$$(n + 3)(n - 1) = n^2 + 2n - 3$$

Answer the difference is always 4 cm^2

13

Seats in a cinema are numbered in rows of 12, as shown. In any 2 by 2 block of seats, multiply the numbers in opposite corners. Prove that the two answers always differ by 12.

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36

$$n, n + 1, n + 12, n + 13$$

$$(n + 1)(n + 12) - n(n + 13) = 12$$

Answer they always differ by 12

14

Take a two digit number, reverse its digits and subtract the smaller number from the larger. Prove that the answer is always a multiple of 9.

$$\text{number } 10a + b, \text{ reversed } 10b + a$$

$$10a + b - (10b + a) = 9a - 9b$$

Answer $9(a - b)$, a multiple of 9

15

Prove that the sum of the squares of three consecutive whole numbers is always 2 more than a multiple of 3.

$$(n - 1)^2 + n^2 + (n + 1)^2$$

$$n^2 - 2n + 1 + n^2 + n^2 + 2n + 1$$

$$= 3n^2 + 2$$

Answer $3n^2 + 2$

16

Lena works out $1 + 2 + 3 + 4 = 10$, $2 + 3 + 4 + 5 = 14$ and $3 + 4 + 5 + 6 = 18$. She says this proves that the sum of four consecutive whole numbers is never a multiple of 4. Explain why this is not a proof. Then prove it.

examples do not cover every number

$$n + (n + 1) + (n + 2) + (n + 3)$$

$$= 4n + 6 = 4(n + 1) + 2$$

Answer $4(n + 1) + 2$